

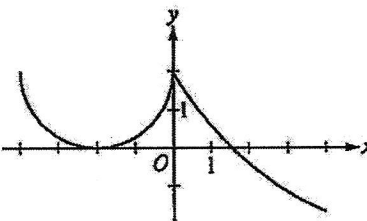
AP[®] CALCULUS AB
2009 SCORING GUIDELINES

Question 6

The derivative of a function f is defined by

$$f'(x) = \begin{cases} g(x) & \text{for } -4 \leq x \leq 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \leq 4 \end{cases}$$

The graph of the continuous function f' , shown in the figure above, has x -intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \leq x \leq 0$ is a semicircle, and $f(0) = 5$.



Graph of f'

- (a) For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.
- (b) Find $f(-4)$ and $f(4)$.
- (c) For $-4 \leq x \leq 4$, find the value of x at which f has an absolute maximum. Justify your answer.

(a) f' changes from decreasing to increasing at $x = -2$ and from increasing to decreasing at $x = 0$. Therefore, the graph of f has points of inflection at $x = -2$ and $x = 0$.

2: $\begin{cases} 1: \text{identifies } x = -2 \text{ or } x = 0 \\ 1: \text{answer with justification} \end{cases}$

(b) $f(-4) = 5 + \int_0^{-4} g(x) dx$
 $= 5 - (8 - 2\pi) = 2\pi - 3$

$$f(4) = 5 + \int_0^4 (5e^{-x/3} - 3) dx$$

$$= 5 + \left. (-15e^{-x/3} - 3x) \right|_{x=0}^{x=4}$$

$$= 8 - 15e^{-4/3}$$

2: $f(-4)$
 1: integral
 1: value
 5: $f(4)$
 1: integral
 1: antiderivative
 1: value

(c) Since $f'(x) > 0$ on the intervals $-4 < x < -2$ and $-2 < x < 3\ln\left(\frac{5}{3}\right)$, f is increasing on the interval $-4 \leq x \leq 3\ln\left(\frac{5}{3}\right)$.

2: $\begin{cases} 1: \text{answer} \\ 1: \text{justification} \end{cases}$

Since $f'(x) < 0$ on the interval $3\ln\left(\frac{5}{3}\right) < x < 4$, f is decreasing on the interval $3\ln\left(\frac{5}{3}\right) \leq x \leq 4$.

Therefore, f has an absolute maximum at $x = 3\ln\left(\frac{5}{3}\right)$.

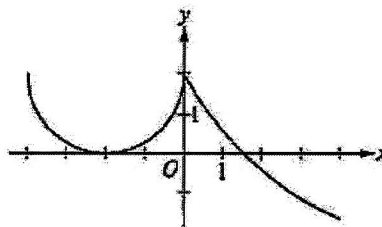
AP Calc AB

Topic 3: Analyzing Graphs of f'

The derivative of a function f is defined by

$$f'(x) = \begin{cases} g(x) & \text{for } -4 \leq x \leq 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \leq 4 \end{cases}$$

The graph of the continuous function f' , shown in the figure above, has x -intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \leq x \leq 0$ is a semicircle, and $f(0) = 5$.

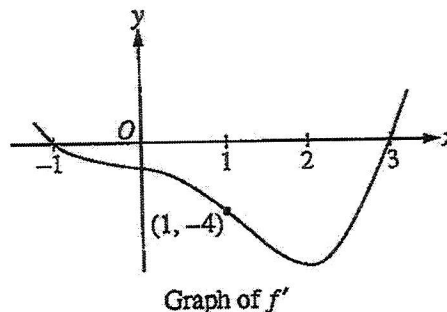
Graph of f'

- For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.
- Find $f(-4)$ and $f(4)$.
- For $-4 \leq x \leq 4$, find the value of x at which f has an absolute maximum. Justify your answer.

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2009 SCORING GUIDELINES (Form B)

Question 5

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f' , the derivative of f , is shown above. The graph of f' crosses the x -axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.



- (a) Write an equation for the line tangent to the graph of g at $x = 1$.
- (b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.
- (c) The second derivative of g is $g''(x) = e^{f(x)}[(f'(x))^2 + f''(x)]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.
- (d) Find the average rate of change of g' , the derivative of g , over the interval $[1, 3]$.

(a) $g(1) = e^{f(1)} = e^2$
 $g'(x) = e^{f(x)}f'(x)$, $g'(1) = e^{f(1)}f'(1) = -4e^2$
 The tangent line is given by $y = e^2 - 4e^2(x - 1)$.

3 : $\begin{cases} 1 : g'(x) \\ 1 : g(1) \text{ and } g'(1) \\ 1 : \text{tangent line equation} \end{cases}$

(b) $g'(x) = e^{f(x)}f'(x)$
 $e^{f(x)} > 0$ for all x
 So, g' changes from positive to negative only when f' changes from positive to negative. This occurs at $x = -1$ only. Thus, g has a local maximum at $x = -1$.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(c) $g''(-1) = e^{f(-1)}[(f'(-1))^2 + f''(-1)]$
 $e^{f(-1)} > 0$ and $f'(-1) = 0$
 Since f' is decreasing on a neighborhood of -1 , $f''(-1) < 0$. Therefore, $g''(-1) < 0$.

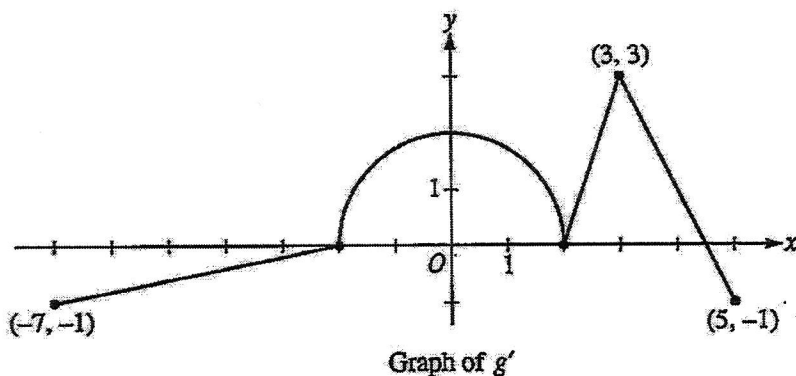
2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $\frac{g'(3) - g'(1)}{3 - 1} = \frac{e^{f(3)}f'(3) - e^{f(1)}f'(1)}{2} = 2e^2$

2 : $\begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$

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2010 SCORING GUIDELINES

Question 5



The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g , consists of a semicircle and three line segments, as shown in the figure above.

- (a) Find $g(3)$ and $g(-2)$.
- (b) Find the x -coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.
- (c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x -coordinate of each critical point of h , where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

(a) $g(3) = 5 + \int_0^3 g'(x) dx = 5 + \frac{\pi \cdot 2^2}{4} + \frac{3}{2} = \frac{13}{2} + \pi$

$g(-2) = 5 + \int_0^{-2} g'(x) dx = 5 - \pi$

- (b) The graph of $y = g(x)$ has points of inflection at $x = 0$, $x = 2$, and $x = 3$ because g' changes from increasing to decreasing at $x = 0$ and $x = 3$, and g' changes from decreasing to increasing at $x = 2$.

(c) $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$

On the interval $-2 \leq x \leq 2$, $g'(x) = \sqrt{4 - x^2}$.

On this interval, $g'(x) = x$ when $x = \sqrt{2}$.

The only other solution to $g'(x) = x$ is $x = 3$.

$h'(x) = g'(x) - x > 0$ for $0 \leq x < \sqrt{2}$

$h'(x) = g'(x) - x \leq 0$ for $\sqrt{2} < x \leq 5$

Therefore h has a relative maximum at $x = \sqrt{2}$, and h has neither a minimum nor a maximum at $x = 3$.

3 : $\begin{cases} 1 : \text{uses } g(0) = 5 \\ 1 : g(3) \\ 1 : g(-2) \end{cases}$

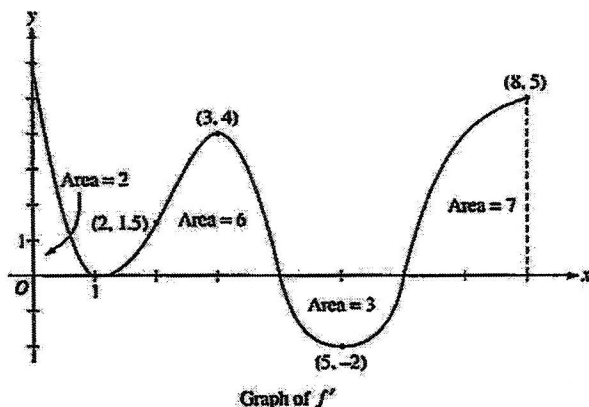
2 : $\begin{cases} 1 : \text{identifies } x = 0, 2, 3 \\ 1 : \text{explanation} \end{cases}$

4 : $\begin{cases} 1 : h'(x) \\ 1 : \text{identifies } x = \sqrt{2}, 3 \\ 1 : \text{answer for } \sqrt{2} \text{ with analysis} \\ 1 : \text{answer for 3 with analysis} \end{cases}$

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2013 SCORING GUIDELINES**

Question 4

The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the closed interval $0 \leq x \leq 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x -axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



- (a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.
- (b) Determine the absolute minimum value of f on the closed interval $0 \leq x \leq 8$. Justify your answer.
- (c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.
- (d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

(a) $x = 6$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a local minimum at $x = 6$.

(b) From part (a), the absolute minimum occurs either at $x = 6$ or at an endpoint.

$$\begin{aligned} f(0) &= f(8) + \int_8^0 f'(x) \, dx \\ &= f(8) - \int_0^8 f'(x) \, dx = 4 - 12 = -8 \end{aligned}$$

$$\begin{aligned} f(6) &= f(8) + \int_8^6 f'(x) \, dx \\ &= f(8) - \int_6^8 f'(x) \, dx = 4 - 7 = -3 \end{aligned}$$

$$f(8) = 4$$

The absolute minimum value of f on the closed interval $[0, 8]$ is -8 .

(c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.

(d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$

$$g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$$

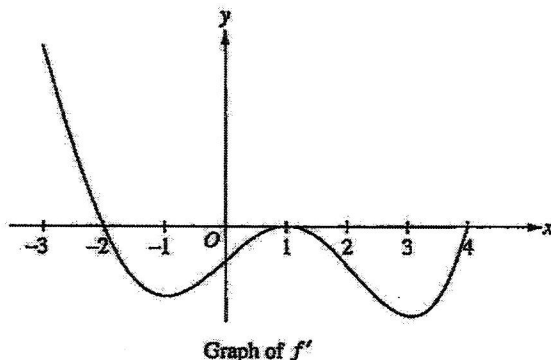
1 : answer with justification

3 : { 1 : considers $x = 0$ and $x = 6$
1 : answer
1 : justification

2 : { 1 : answer
1 : explanation

3 : { 2 : $g'(x)$
1 : answer

The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x -axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



- (a) Find all x -coordinates at which f has a relative maximum. Give a reason for your answer.
- (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.
- (c) Find the x -coordinates of all points of inflection for the graph of f . Give a reason for your answer.
- (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

- (a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$.
 $f'(x)$ changes from positive to negative at $x = -2$.
 Therefore, f has a relative maximum at $x = -2$.

$$2 : \begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$$

- (b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.

$$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$$

- (c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points.

$$2 : \begin{cases} 1 : \text{identifies } x = -1, 1, \text{ and } 3 \\ 1 : \text{reason} \end{cases}$$

The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point.

(d) $f(x) = 3 + \int_1^x f'(t) dt$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$$

$$f(4) = 3 + \int_1^4 f'(t) dt = 3 + (-12) = -9$$

$$\begin{aligned} f(-2) &= 3 + \int_1^{-2} f'(t) dt = 3 - \int_{-2}^1 f'(t) dt \\ &= 3 - (-9) = 12 \end{aligned}$$

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2011 SCORING GUIDELINES (Form B)

Question 4

Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

- (a) Find the x -coordinate of the critical point of f . Determine whether the point is a relative maximum, a relative minimum, or neither for the function f . Justify your answer.
- (b) Find all intervals on which the graph of f is concave down. Justify your answer.
- (c) Given that $f(1) = 2$, determine the function f .

(a) $f'(x) = 0$ at $x = 4$
 $f'(x) > 0$ for $0 < x < 4$
 $f'(x) < 0$ for $x > 4$
 Therefore f has a relative maximum at $x = 4$.

3 : $\begin{cases} 1 : x = 4 \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$

(b) $f''(x) = -x^{-3} + (4 - x)(-3x^{-4})$
 $= -x^{-3} - 12x^{-4} + 3x^{-3}$
 $= 2x^{-4}(x - 6)$
 $= \frac{2(x - 6)}{x^4}$

$f''(x) < 0$ for $0 < x < 6$

The graph of f is concave down on the interval $0 < x < 6$.

3 : $\begin{cases} 2 : f''(x) \\ 1 : \text{answer with justification} \end{cases}$

(c) $f(x) = 2 + \int_1^x (4t^{-3} - t^{-2}) dt$
 $= 2 + \left[-2t^{-2} + t^{-1} \right]_{t=1}^{t=x}$
 $= 3 - 2x^{-2} + x^{-1}$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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2017 SCORING GUIDELINES**

Question 3

(a) $f(-6) = f(-2) + \int_{-2}^{-6} f'(x) dx = 7 - \int_{-6}^{-2} f'(x) dx = 7 - 4 = 3$
 $f(5) = f(-2) + \int_{-2}^5 f'(x) dx = 7 - 2\pi + 3 = 10 - 2\pi$

(b) $f'(x) > 0$ on the intervals $[-6, -2]$ and $(2, 5]$.
 Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.

(c) The absolute minimum will occur at a critical point where $f'(x) = 0$ or at an endpoint.

$$f'(x) = 0 \Rightarrow x = -2, x = 2$$

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is $f(2) = 7 - 2\pi$.

(d) $f''(-5) = \frac{2 - 0}{-6 - (-2)} = -\frac{1}{2}$

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} = 2 \text{ and } \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3} = -1$$

$f''(3)$ does not exist because

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} \neq \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3}$$

3 : $\left\{ \begin{array}{l} 1 : \text{uses initial condition} \\ 1 : f(-6) \\ 1 : f(5) \end{array} \right.$

2 : answer with justification

2 : $\left\{ \begin{array}{l} 1 : \text{considers } x = 2 \\ 1 : \text{answer with justification} \end{array} \right.$

2 : $\left\{ \begin{array}{l} 1 : f''(-5) \\ 1 : f''(3) \text{ does not exist,} \\ \text{with explanation} \end{array} \right.$

AP[®] CALCULUS AB
2018 SCORING GUIDELINES

Question 5

- (a) The average rate of change of f on the interval $0 \leq x \leq \pi$ is

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^\pi - 1}{\pi}.$$

- (b) $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

The slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$ is $e^{3\pi/2}$.

- (c) $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}e^{5\pi/4}$
2π	$e^{2\pi}$

The absolute minimum value of f on $0 \leq x \leq 2\pi$ is $-\frac{1}{\sqrt{2}}e^{5\pi/4}$.

- (d) $\lim_{x \rightarrow \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$$\lim_{x \rightarrow \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$$

By L'Hospital's Rule,

$$\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

1 : answer

2 : $\begin{cases} 1 : f'(x) \\ 1 : \text{slope} \end{cases}$

3 : $\begin{cases} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \frac{\pi}{4}, x = \frac{5\pi}{4} \\ \text{as candidates} \\ 1 : \text{answer with justification} \end{cases}$

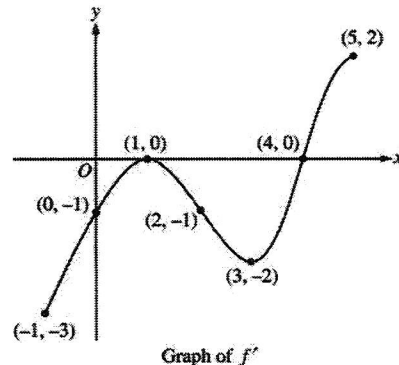
3 : $\begin{cases} 1 : g \text{ is continuous at } x = \frac{\pi}{2} \\ \text{and limits equal } 0 \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0-0] if no limit notation attached to a ratio of derivatives

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2004 SCORING GUIDELINES (Form B)

Question 4

The figure above shows the graph of f' , the derivative of the function f , on the closed interval $-1 \leq x \leq 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.



- (a) Find the x -coordinate of each of the points of inflection of the graph of f . Give a reason for your answer.
- (b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \leq x \leq 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \leq x \leq 5$? Show the analysis that leads to your answers.
- (c) Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

- (a) $x = 1$ and $x = 3$ because the graph of f' changes from increasing to decreasing at $x = 1$, and changes from decreasing to increasing at $x = 3$.

$$2 : \begin{cases} 1 : x = 1, x = 3 \\ 1 : \text{reason} \end{cases}$$

- (b) The function f decreases from $x = -1$ to $x = 4$, then increases from $x = 4$ to $x = 5$.
 Therefore, the absolute minimum value for f is at $x = 4$.
 The absolute maximum value must occur at $x = -1$ or at $x = 5$.

$$4 : \begin{cases} 1 : \text{indicates } f \text{ decreases then increases} \\ 1 : \text{eliminates } x = 5 \text{ for maximum} \\ 1 : \text{absolute minimum at } x = 4 \\ 1 : \text{absolute maximum at } x = -1 \end{cases}$$

$$f(5) - f(-1) = \int_{-1}^5 f'(t) dt < 0$$

Since $f(5) < f(-1)$, the absolute maximum value occurs at $x = -1$.

- (c) $g'(x) = f(x) + xf'(x)$
 $g'(2) = f(2) + 2f'(2) = 6 + 2(-1) = 4$
 $g(2) = 2f(2) = 12$

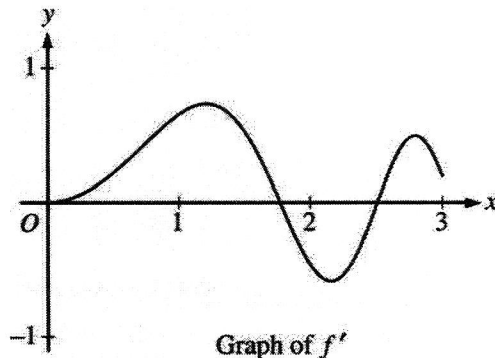
$$3 : \begin{cases} 2 : g'(x) \\ 1 : \text{tangent line} \end{cases}$$

Tangent line is $y = 4(x - 2) + 12$

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2006 SCORING GUIDELINES (Form B)**

Question 2

Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f' , the first derivative of f , given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown above.



- (a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.
- (b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.
- (c) Write an equation for the line tangent to the graph of f at $x = 2$.

(a) On the interval $1.7 < x < 1.9$, f' is decreasing and thus f is concave down on this interval.

(b) $f'(x) = 0$ when $x = 0, \sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}, \dots$
On $[0, 3]$ f' changes from positive to negative only at $\sqrt{\pi}$. The absolute maximum must occur at $x = \sqrt{\pi}$ or at an endpoint.

$$f(0) = 5$$

$$f(\sqrt{\pi}) = f(0) + \int_0^{\sqrt{\pi}} f'(x) dx = 5.67911$$

$$f(3) = f(0) + \int_0^3 f'(x) dx = 5.57893$$

This shows that f has an absolute maximum at $x = \sqrt{\pi}$.

(c) $f(2) = f(0) + \int_0^2 f'(x) dx = 5.62342$

$$f'(2) = e^{-0.5} \sin(4) = -0.45902$$

$$y - 5.623 = (-0.459)(x - 2)$$

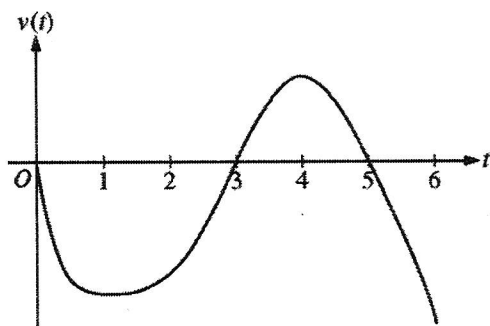
2 : { 1 : answer
1 : reason

3 : { 1 : identifies $\sqrt{\pi}$ and 3 as candidates
- or -
indicates that the graph of f increases, decreases, then increases
1 : justifies $f(\sqrt{\pi}) > f(3)$
1 : answer

4 : { 2 : $f(2)$ expression
1 : integral
1 : including $f(0)$ term
1 : $f'(2)$
1 : equation

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2008 SCORING GUIDELINES

Question 4



Graph of v

A particle moves along the x -axis so that its velocity at time t , for $0 \leq t \leq 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t -axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

- (a) For $0 \leq t \leq 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
- (b) For how many values of t , where $0 \leq t \leq 6$, is the particle at $x = -8$? Explain your reasoning.
- (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
- (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

- (a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$.

$$x(3) = -2 + \int_0^3 v(t) dt = -2 - 8 = -10$$

$$x(6) = -2 + \int_0^6 v(t) dt = -2 - 8 + 3 - 2 = -9$$

Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.

- (b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$.

By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.

- (c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.
- (d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.

$$3 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{considers } \int_0^6 v(t) dt \\ 1 : \text{conclusion} \end{cases}$$

$$3 : \begin{cases} 1 : \text{positions at } t = 3, t = 5, \\ \quad \text{and } t = 6 \\ 1 : \text{description of motion} \\ 1 : \text{conclusion} \end{cases}$$

1 : answer with reason

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

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Solution

(a) horizontal tangent $\Leftrightarrow f'(x) = 0$
 $x = -7, -1, 4, 8$

(b) Relative maxima at $x = -1, 8$ because f' changes from positive to negative at these points

(c) f concave downward $\Leftrightarrow f'$ decreasing
 $(-3, 2), (6, 10)$