AP Calc AB 2019 FRQ's

2019 - FRQ #1

(a) $\int_0^5 \left(20+15\sin\frac{\pi t}{6}\right)dt = 153.458$

About 154 fish enter the lake from $t=0$ to $t=5$.

(b)

$$Avg = \frac{1}{5-0}\int_0^5 L(t)\,dt = 6.059 \text{ fish/hr leaving}$$

An average of 6.059 fish/hour were leaving the lake.

(c) Instantaneous fish in lake: $f(t) = \int_0^t [E(x)-L(x)]\,dt$

Maximum # of fish in lake is when $f'(t)=0$

$f'(t) = E(t) - L(t) = 0 \Rightarrow E(t) = L(t)$

Using technology, $E(t) = L(t)$ at $t = 6.204$

t	$f(t)$
0	0
6.204	135.105
8	80.920

From $0 \le t \le 8$, at $t = 6.204$ hours after midnight is when there is a maximum number of fish in the lake

(d) $f'(t) = E(t) - L(t)$

$f''(5) = E'(5) - L'(5) = -10.723 \text{ fish/hr}^2$

Rate of change in number of fish in lake is decreasing at $t=5$, since $f'(t)$ is decreasing at $t=5$.

AP Calc AB 2019 FRQ's

2019 AP FRQ #2

(a) The average rate of change of velocity in the interval $0.3 \le t \le 2.8$ is zero, and because the function $v(t)$ is continuous and differentiable, by the Mean value theorem there must be at least one point in time between $0.3 \le t \le 2.8$ when instantaneous velocity is zero. ■

(b)
$$\int_0^{2.8} v_p(t)\,dt \approx \frac{0+55}{2}(0.3-0) + \frac{55+(-29)}{2}(1.7-0.3) + \frac{(-29+55)}{2}(2.8-1.7)$$

$$= \underline{\underline{40.75 \text{ meters}}}$$ ■

OR

$$\int_0^{2.8} v_p(t)\,dt \approx \frac{LRAM + RRAM}{2} = \frac{0(0.3)+55(1.4)+(-29)(1.1)+55(1.1)+(-29)(1.4)+55(0.3)}{2}$$

$$= \underline{\underline{40.75 \text{ meters}}}$$

(c) Using technology, $v_Q(t)$ is at least 60 m/hour between

$$1.866 \le t \le 3.519$$

$$\text{Distance} = \int_{1.866}^{3.519} |v_Q(t)|\,dt = \underline{\underline{106.109 \text{ meters}}}$$ ■

(d) $x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t)\,dt = -90 + 135.938 = 45.938$

$x_P(2.8) = x_P(0) + 40.75 = 40.75$

[The distance between the particles is 5.188 meters at $t = 2.8$.] ■

AP Calc AB 2019 FRQ's

2019 FRQ #3

(a) $\int_{-6}^{-2} f(x)\,dx = \int_{-6}^{5} f(x)\,dx + \int_{5}^{-2} f(x)\,dx$

$= \int_{-6}^{5} f(x)\,dx - \int_{-2}^{5} f(x)\,dx$ Note: On interval $[-2, 1]$, the area nets to zero.

$= 7 - \left[\underbrace{\frac{1+3}{2}(1)}_{\substack{\text{Trapezoid} \\ [1,2]}} + \underbrace{(3)(3)}_{\substack{\text{square} \\ [2,5]}} - \underbrace{\frac{\pi(3)^2}{4}}_{\substack{\text{Quarter circle} \\ [2,5]}}\right] = -4 + \frac{9\pi}{4}$ ■

(b) $\int_{3}^{5} (2f'(x)+4)\,dx = 2\int_{3}^{5} f'(x)\,dx + \int_{3}^{5} 4\,dx$

$= 2[f(5) - f(3)] + 4x\Big|_{3}^{5}$

$= 2[0 - (3 - \sqrt{5})] + 8 = 2(-3 + \sqrt{5}) + 8$

$= -6 + 2\sqrt{5} + 8 = \underline{\underline{2 + 2\sqrt{5}}}$ ■

(c) $g(x) = \int_{-2}^{x} f(t)\,dt$ Crit. points where $g'(x) = 0$

$g'(x) = f(x)$ $f(x) = 0$ at $x = -1, 1/2, 5$

x	$g(x)$
-2	0
-1	0.5
$1/2$	-0.25
5	$7+4-9\pi/4 = \underline{\underline{11 - 9\pi/4}}$

Absolute max is at critical point or end point.

Absolute max of $g(x)$ from $[-2, 5]$ is $\underline{\underline{11 - 9\pi/4}}$ ■

(d) $\lim_{x\to 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} = \frac{10 - 3(2)}{1 - \pi/4} = \underline{\underline{\frac{4}{1-\pi/4}}}$ ■

AP Calc AB 2019 FRQs

2019 AP Calc AB FRQ #4

(a) $V = \pi r^2 h$ $\quad r = 1$ ft (constant)

$dV/dt = \pi r^2 \, dh/dt$ $\qquad \left. dh/dt \right|_{h=4ft} = -\frac{1}{10}\sqrt{4} = -\frac{1}{5}$ ft/sec

$\left. dV/dt \right|_{h=4ft} = \pi (1 \text{ ft})^2 \left(-1/5 \text{ ft/sec}\right) = -\frac{\pi}{5}$ ft³/sec ■

(b) $\frac{dh}{dt} = -\frac{1}{10} h^{1/2}$

$\frac{d^2h}{dt^2} = -\frac{1}{20} h^{-1/2} \cdot \frac{dh}{dt} = -\frac{1}{20} h^{-1/2}\left(-\frac{1}{10} h^{1/2}\right) = \frac{1}{200}$

Because d^2h/dt^2 is always positive, the rate of change of height is increasing when height is 3 ft. ■

(c) $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h} \Rightarrow \frac{dh}{\sqrt{h}} = -\frac{1}{10} dt$

$\int h^{-1/2} dh = \int -\frac{1}{10} dt \Rightarrow 2h^{1/2} = -\frac{1}{10} t + C$

Using $h(0) = 5$ ft: $\quad 2\sqrt{5} = C$

So $\quad 2\sqrt{h} = -\frac{1}{10} t + 2\sqrt{5} \Rightarrow \sqrt{h} = -\frac{1}{20} t + \sqrt{5}$

$h = \left(-\frac{1}{20} t + \sqrt{5}\right)^2$ ■

AP Calc AB 2019 FRQs

2019 AP FRQ #5

a.

$$A = \int_0^2 (h(x) - g(x))\,dx$$

$$= \int_0^2 \left(6 - 2(x^2 - 2x + 1)\right)dx - \int_0^2 \left(-2 + 3\cos\left(\tfrac{\pi}{2}x\right)\right)dy$$

$$= \int_0^2 (6 - 2x^2 + 4x - 2)\,dx - \int_0^2 \left(-2 + 3\cos\left(\tfrac{\pi}{2}x\right)\right)dy$$

$$= \int_0^2 (-2x^2 + 4x + 4)\,dx - \int_0^2 \left(-2 + 3\cos\tfrac{\pi}{2}x\right)dx$$

$$= -\frac{2}{3}x^3 + 2x^2 + 4x\Big|_0^2 - \left[-2x + \frac{6}{\pi}\sin\frac{\pi}{2}x\right]_0^2$$

$$= -\frac{16}{3} + 8 + 8 - \left[-4\right] = 20 - 16/3 = \frac{60}{3} - \frac{16}{3} = \frac{44}{3} \;\blacksquare$$

b.

$$V = \int_0^2 A(x)\,dx = \int_0^2 \frac{1}{x+3}\,dx = \ln|x+3|\Big|_0^2 =$$

$$\ln 5 - \ln 3 = \ln(5/3) \;\blacksquare$$

c.

$$V = \pi\int\left[(6 - g(x))^2 - (6 - h(x))^2\right]dx \;\blacksquare$$

AP Calc AB FRQ #6 2019

(a) $h'(2) = 2/3$ (from tangent line equation)

(b) $a(x) = 3x^3 h(x) \Rightarrow a'(x) = 3x^3 h'(x) + 9x^2 h(x)$

$a'(2) = 3(2)^3 h'(2) + 9(2)^2 h(2) = 24(2/3) + 36(4) = 160$

(c) $h(x) = \dfrac{x^2-4}{1-[f(x)]^3}$ for $x \neq 2$

If $\lim\limits_{x\to 2} h(x)$ is solvable w/ L'Hopital, then $1-[f(2)]^3 = 0$

$\Rightarrow \underline{\underline{f(2) = 1}}$

Because $h(x)$ is differentiable, $h(x)$ is also continuous, so:

$\lim\limits_{x\to 2} h(x) = h(2) = 4$, so by L'Hopital:

$$\lim_{x\to 2} \frac{x^2-4}{1-[f(x)]^3} = \lim_{x\to 2} \frac{2x}{-3f(x)^2 f'(x)} = \frac{2(2)}{-3f(2)^2 f'(2)} = 4$$

Solving for $f'(2)$, we get $\underline{\underline{f'(2) = -1/3}}$

(d) Because $g(x) \le k(x) \le h(x)$ for $1 < x < 3$, then

$\lim\limits_{x\to 2} g(x) \le \lim\limits_{x\to 2} k(x) \le \lim\limits_{x\to 2} h(x)$

Because $g(x)$ and $h(x)$ are differentiable and therefore continuous, $\lim\limits_{x\to 2} g(x) = g(2) = 4$, and $\lim\limits_{x\to 2} h(x) = h(2) = 4$.

So, by substitution: $4 \le \lim\limits_{x\to 2} k(x) \le 4 \Rightarrow \lim\limits_{x\to 2} k(x) = 4$

Also, $g(2) \le k(2) \le h(2) \Rightarrow 4 \le k(2) \le 4 \Rightarrow k(2) = 4$.

Because $k(2) = \lim\limits_{x\to 2} k(x) = 4$, by definition of continuity we know $k(x)$ is continuous at $x = 2$