AP Stats Topic 3: Gathering Data

Every Monday a local radio station gives coupons away to 50 people who correctly answer a question about a news fact from the previous day's newspaper. The coupons given away are numbered from 1 to 50, with the first person receiving coupon 1, the second person receiving coupon 2, and so on, until all 50 coupons are given away. On the following Saturday, the radio station randomly draws numbers from 1 to 50 and awards cash prizes to the holders of the coupons with these numbers. Numbers continue to be drawn without replacement until the total amount awarded first equals or exceeds $300. If selected, coupons 1 through 5 each have a cash value of $200, coupons 6 through 20 each have a cash value of $100, and coupons 21 through 50 each have a cash value of $50.

(a) Explain how you would conduct a simulation using the random number table provided below to estimate the distribution of the number of prize winners each week.

(b) Perform your simulation 3 times. (That is, run 3 trials of your simulation.) Start at the leftmost digit in the first row of the table and move across. Make your procedure clear so that someone can follow what you did. You must do this by marking directly on or above the table. <u>Report the number of winners in each of your 3 trials</u>.

72749	13347	65030	26128	49067	02904	49953	74674	94617	13317
81638	36566	42709	33717	59943	12027	46547	61303	46699	76423
38449	46438	91579	01907	72146	05764	22400	94490	49833	09258

Part (a):

1. **Scheme:** Obtain a two-digit random number from the random number table. If it is between 01 and 50, use it to represent the selected ticket. Ignore numbers 00 and 51 – 99.
2. **Stopping Rule:** Determine the amount of the prize associated with the chosen ticket, and add this amount to the total amount awarded so far. If the total amount awarded so far is less than $300, repeat this process.
3. **Count:** Note the total number of winners.
4. **Non-Replacement:** Ignore any ticket number that has already been awarded a prize in this trial.

Repeats steps 1 – 4 above a large number of times.

Note: It is OK to also devise a scheme that uses 2 two-digit numbers to represent each ticket (for example, 01 and 51 both representing ticket 1; 02 and 52 both representing ticket 2; etc.) that also addresses the issue of assigning 2 two-digit numbers to each coupon correctly.

Part (b):

Solution will depend on answer to part (a).

For example, using scheme above:

Trial 1			Trial 2			Trial 3		
					Total so far			Total so far
72	ignore	0	02	200	200	06	100	100
74	ignore	0	61	ignore	200	70	ignore	100
91	ignore	0	28	50	250	29	50	150
33	50	50	48	50	300	04	200	350
47	50	100						
65	ignore	100	Total number of winners: 3			Total number of winners: 3		
03	200	300						
Total number of winners: 3								

Students should perform 3 trials. You will have to look at each student response carefully. Some will continue on 1st row, some will use 2nd row for second trial, etc.

A manufacturer of toxic pesticide granules plans to use a dye to color the pesticide so that birds will avoid eating it. A series of experiments will be designed to find colors or patterns that three bird species (blackbirds, starlings, and geese) will avoid eating. Representative samples of birds will be captured to use in the experiments, and the response variable will be the amount of time a hungry bird will avoid eating food of a particular color or pattern.

(a) Previous research has shown that male birds do not avoid solid colors. However, it is possible that males might avoid colors displayed in a pattern, such as stripes. In an effort to prevent males from eating the pesticide, the following two treatments are applied to the pesticide granules.

Treatment 1: A red background with narrow blue stripes

Treatment 2: A blue background with narrow red stripes

To increase the power of detecting a difference in the two treatments in the analysis of the experiment, the researcher decided to block on the three species of birds (blackbirds, starlings, and geese). Assuming there are 100 birds of each of the three species, explain how you would assign birds to treatments in such a block design.

(b) Other than blocking, what could the researcher do to increase the power of detecting a difference in the two treatments in the analysis of the experiment? Explain how your approach would increase the power.

Solution

Part (a):

Form three blocks based on the species of bird (blackbirds, starlings, and geese) to accomplish the goal of blocking to create groups of homogeneous experimental units. Within each of the three blocks, carry out a completely randomized design by randomly assigning the birds within each block to one of the two treatments. Within block 1, each bird of a particular species (let's say the blackbirds) will be tagged with a unique random number using a random number generator on a calculator, statistical software, or a random number table. The random numbers will be sorted from lowest to highest. The birds with the lowest 50 numbers in the ordered list will receive treatment 1 (red background with narrow blue stripes). The birds with the highest 50 numbers will receive treatment 2 (blue background with narrow red stripes). This method of randomization should be repeated in the other two blocks.

Part (b):

To increase power (other than by blocking), the researcher could increase the sample size. This reduces the standard error of the sampling distribution. With a smaller standard error, a test is more likely to be able to detect a difference in results from the two treatments, if such a difference exists.

High cholesterol level in people can be reduced by exercise or by drug treatment. A pharmaceutical company has developed a new cholesterol-reducing drug. Researchers would like to compare its effects to the effects of the cholesterol-reducing drug that is currently available on the market. Volunteers who have a history of high cholesterol and who are currently not on medication will be recruited to participate in a study.

(a) Explain how you would carry out a completely randomized experiment for the study.

(b) Describe an experimental design that would improve the design in (a) by incorporating blocking.

(c) Can the experimental design in (b) be carried out in a double blind manner? Explain.

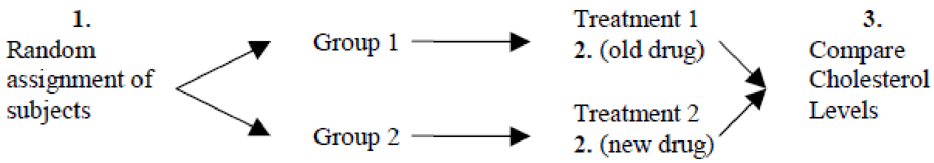
Solution

(a) Describes an experimental design that includes:

1. Random assignment of volunteers to the treatment groups
2. Identification of treatment groups as old drug and new drug
3. Indication that a comparison or measurement of cholesterol levels should be made

OR

The student may give a detailed diagram that addresses the three parts:



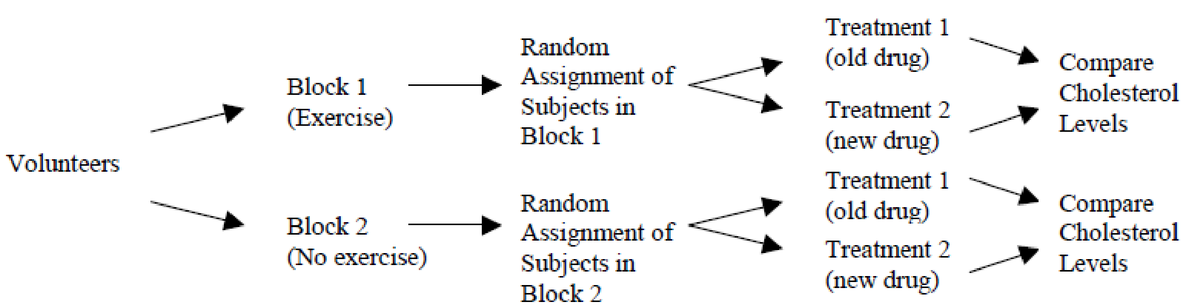
Note: In part (a), it is incorrect to use the terminology "treatment" and "placebo" for the treatment groups. It is considered correct to use "old drug" and "new drug", and "placebo," if a third group is used, for the treatment groups.

(b) Describes an experimental design that includes:

1. Creating blocks based on level of exercise or cholesterol level, or creating blocks using age, diet, gender, or any other factor plausibly related to cholesterol level **with explanation** (i.e., block on gender because males and females may respond differently)
2. Random assignment of subjects to treatments within blocks

OR

The student may give a detailed diagram that addresses the two parts as long as the blocking factor is described.



Note: No credit will be given in part (b) if a student does not use blocking in his/her design even though they randomize correctly.

Note: Crossover designs or matched-pairs designs that incorporate the idea of blocking are acceptable.

(c) Clearly explains a double blind experiment—neither the subjects nor those administering the drugs or monitoring results know which of the two drugs is being used.

An answer of yes without explanation receives no credit.

An answer of no could receive credit if the design described in part (b) does not allow for double-blinding.

The dentists in a dental clinic would like to determine if there is a difference between the number of new cavities in people who eat an apple a day and in people who eat less than one apple a week. They are going to conduct a study with 50 people in each group.

Fifty clinic patients who report that they routinely eat an apple a day and 50 clinic patients who report that they eat less than one apple a week will be identified. The dentists will examine the patients and their records to determine the number of new cavities the patients have had over the past two years. They will then compare the number of new cavities in the two groups.

a. Why is this an observational study and not an experiment?

b. Explain the concept of confounding in the context of this study. Include an example of a possible confounding variable.

c. If the mean number of new cavities for those who ate an apple a day was statistically significantly smaller than the mean number of new cavities for those who ate less than one apple a week, could one conclude that the lower number of new cavities can be attributed to eating an apple a day? Explain.

Solution
This problem has three parts.

a. The student can appeal to any of three reasons in judging this study not an experiment:

   1. there is no random assignment of subjects to treatments;
   2. there are no treatments imposed;
   3. existing data is being used.

b. Two variables are confounded if their effect on the number of new cavities cannot be distinguished from one another. The student must mention not only that the confounding variables may affect the outcome but that they have differential effects within the two groups. For instance: confounding would occur if patients who eat an apple a day differ from those who eat less than one apple a week **on some variable that is related to dental health**. In this example, diet or general level of health are examples of what might be confounding variables. For example, it is possible that people who eat an apple a day are more nutrition conscious and have a more healthy diet in general than those who eat one or fewer apples per week, and this might explain the observed difference in dental health.

Note:
There are many possible examples of confounding variables. Any reasonable example of a confounding variable is acceptable, as long as a good explanation is given and the connection between the confounding variable and group membership is clear. Lack of a definition here can be rectified by a response in (c) that demonstrates a clear understanding of the concept of confounding variable.

c. No, because it is not an experiment, and cause-and-effect conclusions cannot be drawn from an observational study.

   OR

   No, because there are possible confounding variables.

People with acrophobia (fear of heights) sometimes enroll in therapy sessions to help them overcome this fear. Typically, seven or eight therapy sessions are needed before improvement is noticed. A study was conducted to determine whether the drug D-cycloserine, used in combination with fewer therapy sessions, would help people with acrophobia overcome this fear.

Each of 27 people who participated in the study received a pill before each of two therapy sessions. Seventeen of the 27 people were randomly assigned to receive a D-cycloserine pill, and the remaining 10 people received a placebo. After the two therapy sessions, none of the 27 people received additional pills or therapy. Three months after the administration of the pills and the two therapy sessions, each of the 27 people was evaluated to see if he or she had improved.

(a) Was this study an experiment or an observational study? Provide an explanation to support your answer.

(b) When the data were analyzed, the D-cycloserine group showed statistically significantly more improvement than the placebo group did. Based on this result, would the researchers be justified in concluding that the D-cycloserine pill and two therapy sessions are as beneficial as eight therapy sessions without the pill? Justify your answer.

(c) A newspaper article that summarized the results of this study did not explain how it was determined which people received D-cycloserine and which received the placebo. Suppose the researchers allowed the therapists to choose which people received D-cycloserine and which received the placebo, and no randomization was used. Explain why such a method of assignment might lead to an incorrect conclusion.

Solution

Part (a):

The study was an experiment because treatments (D-cycloserine or placebo) were imposed by the researchers on the people with acrophobia.

Part (b):

No, the experiment was designed to compare the D-cycloserine group with a control group that received the placebo. The researchers can conclude that the D-cycloserine pill and two therapy sessions show significantly more improvement than a placebo and two therapy sessions. However, there is no basis for comparison with another group of people with acrophobia who received eight therapy sessions and no pill.

Part (c):

One example is that if the therapists were allowed to choose who received the placebo and who received D-cycloserine, they might assign the people with more severe acrophobia to one of the groups and the people with less severe acrophobia to the other group. Thus, the improvement after only two therapy sessions could be related to the initial severity of the acrophobia rather than to the effects of D-cycloserine.

Before beginning a unit on frog anatomy, a seventh-grade biology teacher gives each of the 24 students in the class a pretest to assess their knowledge of frog anatomy. The teacher wants to compare the effectiveness of an instructional program in which students physically dissect frogs with the effectiveness of a different program in which students use computer software that only simulates the dissection of a frog. After completing one of the two programs, students will be given a posttest to assess their knowledge of frog anatomy. The teacher will then analyze the changes in the test scores (score on posttest minus score on pretest).

(a) Describe a method for assigning the 24 students to two groups of equal size that allows for a statistically valid comparison of the two instructional programs.

(b) Suppose the teacher decided to allow the students in the class to select which instructional program on frog anatomy (physical dissection or computer simulation) they prefer to take, and 11 students choose actual dissection and 13 students choose computer simulation. How might that self-selection process jeopardize a statistically valid comparison of the changes in the test scores (score on posttest minus score on pretest) for the two instructional programs? Provide a specific example to support your answer.

Solution

Part (a) (completely randomized design):

Each student will be assigned a unique random number using a random number generator on a calculator, statistical software, or a random number table. The assigned numbers will be listed in ascending order. The students with the lowest 12 numbers in the ordered list will receive the instructional program that requires physically dissecting frogs. The students with the highest 12 numbers will receive the instructional program that uses computer software to simulate the dissection of a frog.

Part (a) *alternative* (randomized block design):

Students will be paired or placed into blocks of size two, based on having similar pretest scores. So, the first block will contain the two students with the two lowest pretest scores, the second block will contain the two students with the third- and fourth-lowest pretest scores, and so on, with the last block containing the two students with the two highest pretest scores. In each block, the students will be assigned a unique random number using a random number generator on a calculator, statistical software, or a random number table. The student in each block with the lower random number will receive the instructional program that requires physically dissecting frogs, and the student with the higher random number will receive the instructional program that uses computer software to simulate the dissection of a frog.

Part (b):

By not randomizing and allowing the students to self-select, there is a potential for changes to occur in the differences between pretest and posttest scores for a particular group because of the characteristics of students who choose a particular instructional method, not because of the instructional method itself. For example, suppose frog-loving students already know a lot about frog anatomy; one would therefore expect these students to be less likely to show a large change between the pretest and posttest scores. Suppose the frog-loving students tend to select the computer simulation method (perhaps because they do not like the notion of dissecting the frogs they love). The possible low change between pretest and posttest scores for the computer simulation group might then be attributed to the students' already knowing a lot about frog anatomy beforehand, not to the instructional method itself. The frog dissection group might see a larger change in scores because the students entering this group are those with the lower pretest scores (less prior knowledge) and who are thus more likely to show greater improvement between pretest and posttest scores.

A preliminary study conducted at a medical center in St. Louis has shown that treatment with small, low-intensity magnets reduces the self-reported level of pain in polio patients. During each session, a patient rested on an examining table in the doctor's office while the magnets, embedded in soft pads, were strapped to the body at the site of pain. Sessions continued for several weeks, after which pain reduction was measured.

A new study is being designed to investigate whether magnets also reduce pain in patients suffering from herniated disks in the lower back. One hundred male patients are available for the new study.

(a) Describe an appropriate design for the new study. Your discussion should briefly address treatments used, methods of treatment assignment, and what variables would be measured. Do not describe how the data would be analyzed.

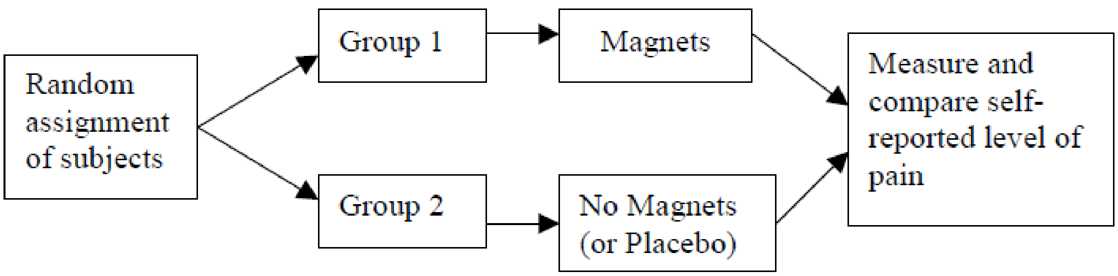
(b) Would you modify the design above if, instead of 100 male patients, there were 50 male and 50 female patients available for the study? If so, how would you modify your design? If not, why not?

Solution

Part (a):

1. Two treatments: magnets and no magnets (or magnets and placebo). Subjects in the no magnet group would be handled in the same way as the magnet group, but there would be no magnets embedded in the pads used.
2. There must be random assignment of subjects to treatments (or treatments to subjects). How the randomization would be carried out does not need to be specified, but it must be clear *what* is being randomized.
3. Variable measured: Self-reported level of pain or reduction in pain.

The design may be described by a diagram, but the treatments and the variable measured must be included and the randomization must be very clear.

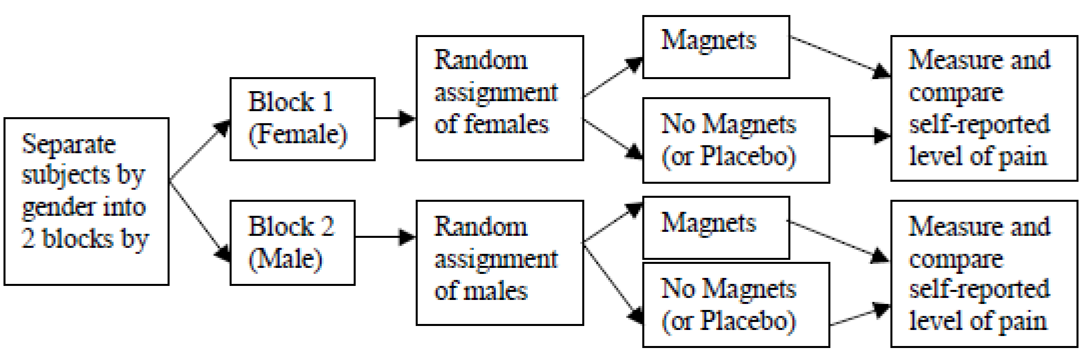


Part (b): Either one of the following approaches is acceptable.

1. Saying yes and indicating how they would alter the design: Separating the subjects into the two gender groups and then randomizing subjects to treatments within each group. This may also be described using a diagram, as shown below, but the blocking factor and randomization must be clearly indicated.

   OR

2. Saying no and describing why. For example, indicating that the randomization in (a) should equalize the effects of gender in the two groups or assuming gender does not have a strong effect and since the sample size is large
   OR
   providing a good explanation for *why* gender does not have a strong differential effect on the outcome.



An administrator at a large university wants to conduct a survey to estimate the proportion of students who are satisfied with the appearance of the university buildings and grounds. The administrator is considering three methods of obtaining a sample of 500 students from the 70,000 students at the university.

(a) Because of financial constraints, the first method the administrator is considering consists of taking a convenience sample to keep the expenses low. A very large number of students will attend the first football game of the season, and the first 500 students who enter the football stadium could be used as a sample. Why might such a sampling method be biased in producing an estimate of the proportion of students who are satisfied with the appearance of the buildings and grounds?

(b) Because of the large number of students at the university, the second method the administrator is considering consists of using a computer with a random number generator to select a simple random sample of 500 students from a list of 70,000 student names. Describe how to implement such a method.

(c) Because stratification can often provide a more precise estimate than a simple random sample, the third method the administrator is considering consists of selecting a stratified random sample of 500 students. The university has two campuses with male and female students at each campus. Under what circumstance(s) would stratification by campus provide a more precise estimate of the proportion of students who are satisfied with the appearance of the university buildings and grounds than stratification by gender?

Solution

Part (a):

The first 500 students who enter the football stadium were not likely to be representative of the population of all students at the university. In other words, these 500 students were likely to differ systematically from the population with regard to many variables. For example, these 500 students might have more school pride than the population of students as a whole, which might be related to their opinions about the appearance of university buildings and grounds. Perhaps their school pride is related to having more positive opinions about the appearance of university buildings and grounds, in which case the sample proportion of students who were satisfied would be biased toward overestimating the population proportion of students who were satisfied.

Part (b):

Obtain a list of all 70,000 students at the university. Assign an identification number from 1 to 70,000 to each student.

Then use a computer to generate 500 random integers between 1 and 70,000 without replacement. The students whose ID numbers correspond to those numbers were then selected for the sample.

Part (c):

Stratifying by campus would be more advantageous than stratifying by gender provided that opinions about appearance of university buildings and grounds between the two campuses differ more than the opinions about appearance of university buildings and grounds between the two genders.

A large regional real estate company keeps records of home sales for each of its sales agents. Each month, the company publishes the sales volume for each agent. Monthly sales volume is defined as the total sales price of all homes sold by the agent during a month. The figure below displays the cumulative relative frequency plot of the most recent monthly sales volume (in hundreds of thousands of dollars) for these agents.

(a) In the context of this question, explain what information is conveyed by the circled point.

(b) What proportion of sales agents achieved monthly sales volumes between \$700,000 and \$800,000 ?

(c) For values between 10 and 11 on the horizontal axis, the cumulative relative frequency plot is flat. In the context of this question, explain what this means.

(d) A bonus is to be given to 20 percent of the sales agents. Those who achieved the highest monthly sales volume during the preceding month will receive a bonus. What is the minimum monthly sales volume an agent must have achieved to qualify for the bonus?

Solution

Part (a):

This point indicates that 40 percent of the sales agents at this real estate company had sales volume of \$300,000 or less in the month shown.

Part (b):

Eighty percent of the sales agents had sales volume of \$800,000 or less and 70 percent of the sales agents had sales volume of \$700,000 or less. Thus, $0.8 - 0.7 = 0.1$ or 10 percent of the sales agents achieved monthly sales volumes greater than \$700,000 and not exceeding \$800,000.

Part (c):

There were no agents whose monthly sales volume was between \$1,000,000 and \$1,100,000.

Part (d):

The 80^{th} percentile for the distribution of monthly sales volume by these agents during the preceding month is \$800,000. Therefore, an agent making more than \$800,000 will be in the top 20 percent.

2. Researchers are investigating the effectiveness of using a fungus to control the spread of an insect that destroys trees. The researchers will create four different concentrations of fungus mixtures: 0 milliliters per liter (ml/L) 1.25 milliliters per liter, 2.5 milliliters per liter, and 3.75 milliliters per liter. An equal number of the insects will be placed into 20 individual containers. The group of insects in each container will be sprayed with one of the four mixtures, and the researchers will record the number of insects that are still alive in each container one week after spraying.

(a) Identify the treatments, experimental units, and response variable of the experiment. Treatments:

Experimental units:

Response variable:

(b) Does the experiment have a control group? Explain your answer.

(c) Describe how the treatments can be randomly assigned to the experimental units so that each treatment has the same number of units.

a.

Treatments: 0, 1.25, 2.5, and 3.75 ml/L

Experimental Units: The 20 individual containers

Response Variable: the number of insects alive one week after spraying.

b. This experiment does have a control group: the containers receiving 0 ml/L of fungus mixture.

c. I would assign each container a number between 1 and 20. Using two digit random numbers, I would generate 5 random numbers, skipping repeats. The containers with these five numbers would receive the first treatment. I would repeat this process for the remaining containers until each treatment was randomly applied to five containers.