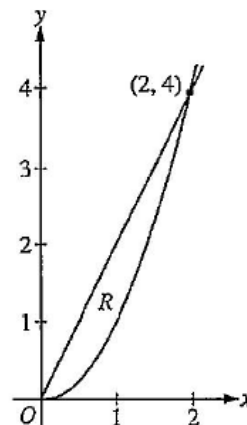


AP Calculus AB

Topic 1: Area and Volume

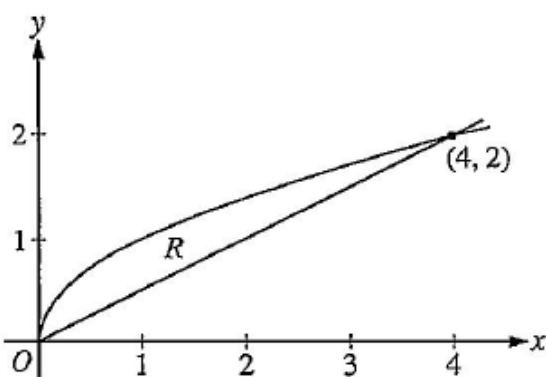
Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

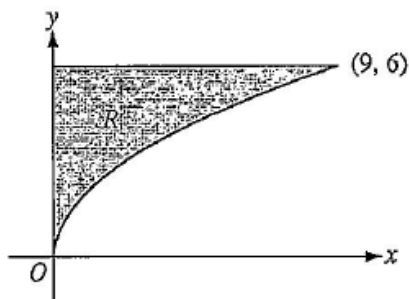
- (a) Find the area of R .
- (b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.
- (c) Another solid has the same base R . For this solid, the cross sections perpendicular to the y -axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.



Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.

- (a) Find the area of R .
- (b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are squares. Find the volume of this solid.
- (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.



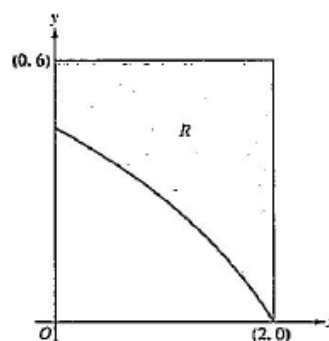


Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y -axis, as shown in the figure above.

- (a) Find the area of R .
- (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.
- (c) Region R is the base of a solid. For each y , where $0 \leq y \leq 6$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose height is 3 times the length of its base in region R . Write, but do not evaluate, an integral expression that gives the volume of the solid.

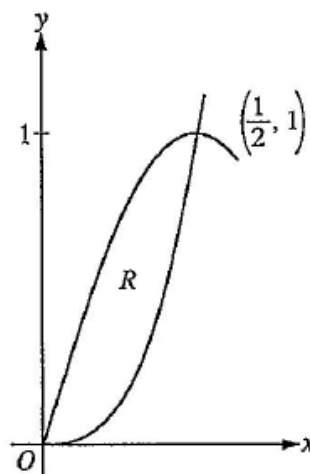
In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4 \ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.
- (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of the solid.

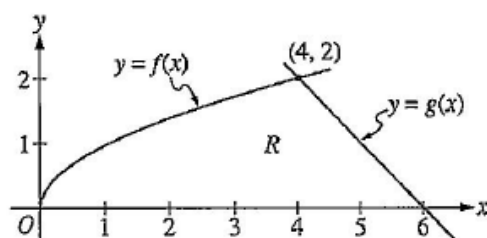


Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

- (a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.
- (b) Find the area of R .
- (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

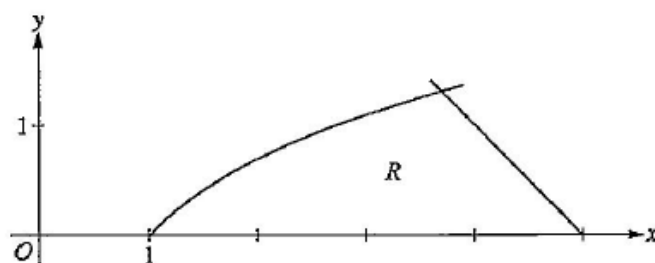


The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x -axis and the graphs of f and g , as shown in the figure above.



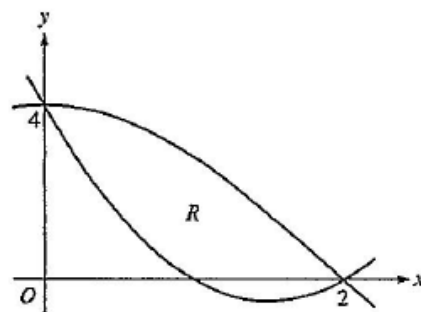
- (a) Find the area of R .
- (b) The region R is the base of a solid. For each y , where $0 \leq y \leq 2$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.
- (c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g . Find the coordinates of point P .

Let R be the region in the first quadrant bounded by the x -axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



- Find the area of R .
- Region R is the base of a solid. For the solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
- The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .

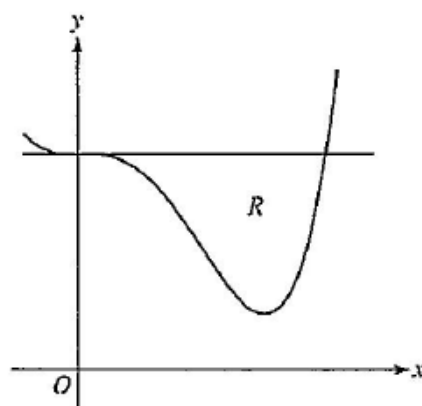
Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g , as shown in the figure above.



- (a) Find the area of R .
- (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.
- (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

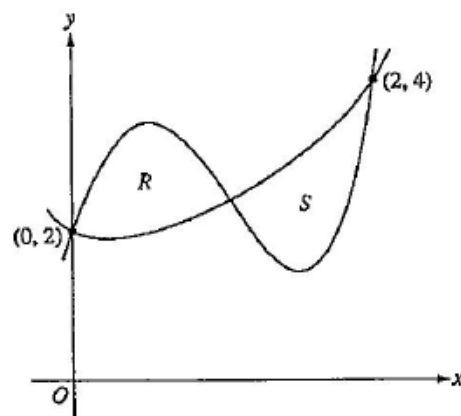
Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

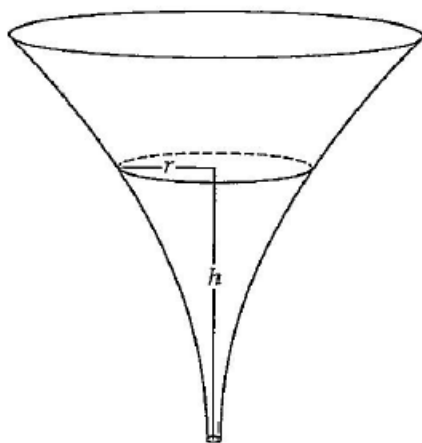
- Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
- Region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is an isosceles right triangle with a leg in R . Find the volume of the solid.
- The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k .



Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

- Find the sum of the areas of regions R and S .
- Region S is the base of a solid whose cross sections perpendicular to the x -axis are squares. Find the volume of the solid.
- Let h be the vertical distance between the graphs of f and g in region S . Find the rate at which h changes with respect to x when $x = 1.8$.





The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.

- (a) Find the average value of the radius of the funnel.
- (b) Find the volume of the funnel.
- (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?