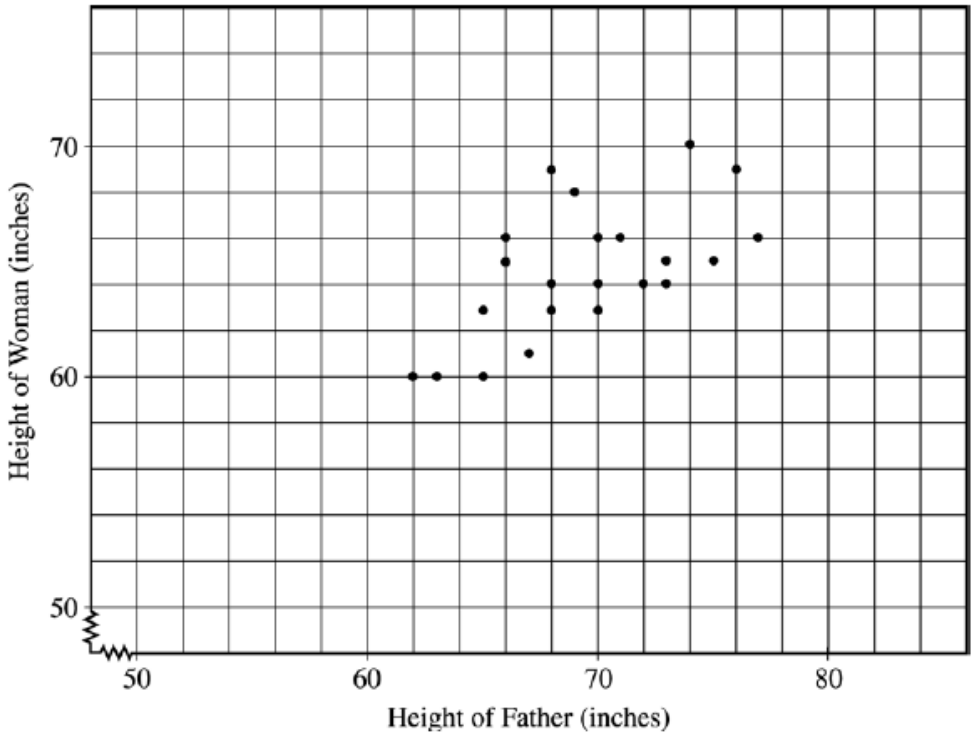
AP FRQ Review – Mr. Rich Name:_____ Per:_____ Seat:____

AP Stats Topic 2: Least Squares Regression

Each of 25 adult women was asked to provide her own height (y), in inches, and the height (x), in inches, of her father. The scatterplot below displays the results. Only 22 of the 25 pairs are distinguishable because some of the (x,y) pairs were the same. The equation of the least squares regression line is $\hat{y} = 35.1 + 0.427x$.



(a) Draw the least squares regression line on the scatterplot above.

(b) One father's height was $x = 67$ inches and his daughter's height was $y = 61$ inches. Circle the point on the scatterplot above that represents this pair and draw the segment on the scatterplot that corresponds to the residual for it. Give a numerical value for the residual.

(c) Suppose the point $x = 84$, $y = 71$ is added to the data set. Would the slope of the least squares regression line increase, decrease, or remain about the same? Explain.

(Note: No calculations are necessary to answer this question.)

Would the correlation increase, decrease, or remain about the same? Explain.

(Note: No calculations are necessary to answer this question.)

Windmills generate electricity by transferring energy from wind to a turbine. A study was conducted to examine the relationship between wind velocity in miles per hour (mph) and electricity production in amperes for one particular windmill. For the windmill, measurements were taken on twenty-five randomly selected days, and the computer output for the regression analysis for predicting electricity production based on wind velocity is given below. The regression model assumptions were checked and determined to be reasonable over the interval of wind speeds represented in the data, which were from 10 miles per hour to 40 miles per hour.

Predictor	Coef	SE Coef	T	P
Constant	0.137	0.126	1.09	0.289
Wind velocity	0.240	0.019	12.63	0.000

S = 0.237 R-Sq = 0.873 R-Sq (adj) = 0.868

(a) Use the computer output above to determine the equation of the least squares regression line. Identify all variables used in the equation.

(b) How much more electricity would the windmill be expected to produce on a day when the wind velocity is 25 mph than on a day when the wind velocity is 15 mph? Show how you arrived at your answer.

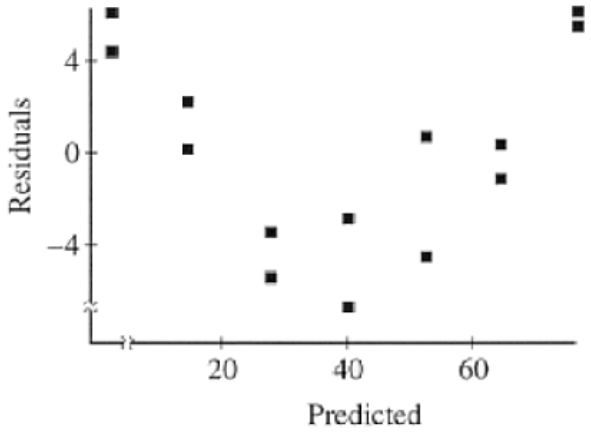
(c) What proportion of the variation in electricity production is explained by its linear relationship with wind velocity?

(d) Is there statistically convincing evidence that electricity production by the windmill is related to wind velocity? Explain.

In a study of the application of a certain type of weed killer, 14 fields containing large numbers of weeds were treated. The weed killer was prepared at seven different strengths by adding 1, 1.5, 2, 2.5, 3, 3.5, or 4 teaspoons to a gallon of water. Two randomly selected fields were treated with each strength of weed killer. After a few days, the percentage of weeds killed on each filed was measured. The computer output obtained from fitting a least squares regression line to the data is shown below. A plot of the residuals is provided as well.

Dependent variable is: percent killed
R squared = 97.2% R squared (adjusted) = 96.9%
$s = 4.505$ with $14 - 2 = 12$ degrees of freedom

Source	Sum of Squares	df	Mean Square	F-ratio
Regression	8330.16	1	8330.16	410
Residual	243.589	12	20.2990	
Variable	**Coefficient**	**s.e. of Coeff**	**t-ratio**	**Prob**
Constant	-20.5893	3.242	-6.35	≤ 0.0001
No. Teaspoons	24.3929	1.204	20.3	≤ 0.0001



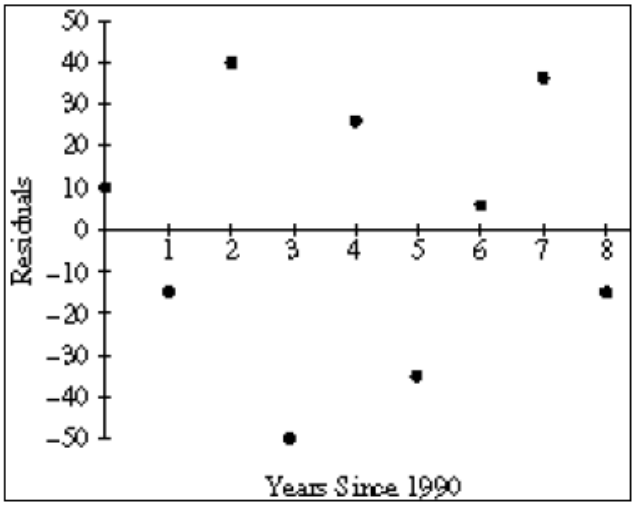
(a) What is the equation of the least squares regression line given by this analysis? Define any variables used in this equation.

(b) If someone uses this equation to predict the percentage of weeds killed when 2.6 teaspoons of weed killer are used, which of the following would you expect?

- ❍ The prediction will be too large.
- ❍ The prediction will be too small.
- ❍ A prediction cannot be made based on the information given on the computer output.

Explain your reasoning.

Lydia and Bob were searching the Internet to find information on air travel in the United States. They found data on the number of commercial aircraft flying in the United States during the years 1990-1998. The dates were recorded as years since 1990. Thus, the year 1990 was recorded as year 0. They fit a least squares regression line to the data. The graph of the residuals and part of the computer output for their regression are given below.



Predictor	Coef	Stdev	t-ratio	p
Constant	2939.93	20.55	143.09	0.000
Years	233.517	4.316	54.11	0.000
s = 33.43				

a. Is a line an appropriate model to use for these data? What information tells you this?

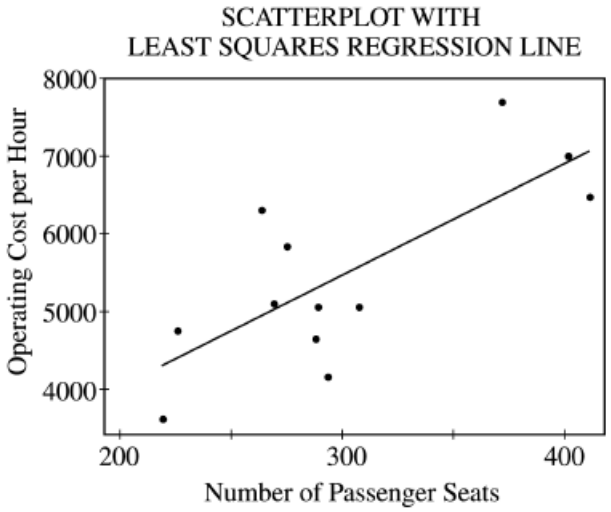
b. What is the value of the slope of the least squares regression line?
Interpret the slope in the context of this situation.

c. What is the value of the intercept of the least squares regression line?
Interpret the intercept in the context of this situation.

d. What is the predicted number of commercial aircraft flying in 1992 ?

e. What was the actual number of commercial aircraft flying in 1992 ?

Commercial airlines need to know the operating cost per hour of flight for each plane in their fleet. In a study of the relationship between operating cost per hour and number of passenger seats, investigators computed the regression of operating cost per hour on the number of passenger seats. The 12 sample aircraft used in the study included planes with as few as 216 passenger seats and planes with as many as 410 passenger seats. Operating cost per hour ranged between $3,600 and $7,800. Some computer output from a regression analysis of these data is shown below.



Predictor	Coef	StDev	T	P
Constant	1136	1226	0.93	0.376
Seats	14.673	4.027	3.64	0.005
S = 845.3	R-Sq = 57.0%		R-Sq (adj) = 52.7%	

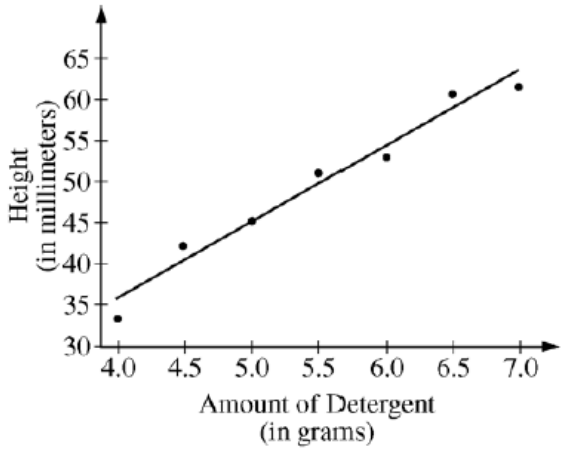
(a) What is the equation of the least squares regression line that describes the relationship between operating cost per hour and number of passenger seats in the plane? Define any variables used in this equation.

(b) What is the value of the correlation coefficient for operating cost per hour and number of passenger seats in the plane? Interpret this correlation.

(c) Suppose that you want to describe the relationship between operating cost per hour and number of passenger seats in the plane for planes only in the range of 250 to 350 seats. Does the line shown in the scatterplot still provide the best description of the relationship for data in this range? Why or why not?

A manufacturer of dish detergent believes the height of soapsuds in the dishpan depends on the amount of detergent used. A study of the suds' heights for a new dish detergent was conducted. Seven pans of water were prepared. All pans were of the same size and type and contained the same amount of water. The temperature of the water was the same for each pan. An amount of dish detergent was assigned at random to each pan, and that amount of detergent was added to the pan. Then the water in the dishpan was agitated for a set amount of time, and the height of the resulting suds was measured.

A plot of the data and the computer output from fitting a least squares regression line to the data are shown below.



Predictor	Coef	SE Coef	T	P
Constant	−2.679	4.222	−0.63	0.554
Amount	9.5000	0.7553	12.58	0.000

S = 1.99821 R–Sq = 96.9% R–Sq(adj) = 96.3%

(a) Write the equation of the fitted regression line. Define any variables used in this equation.

(b) Note that $s = 1.99821$ in the computer output. Interpret this value in the context of this study.

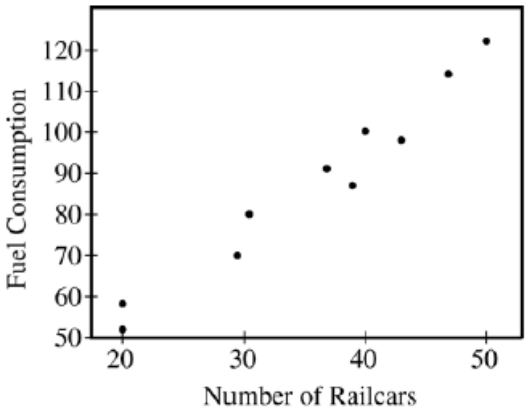
(c) Identify and interpret the standard error of the slope.

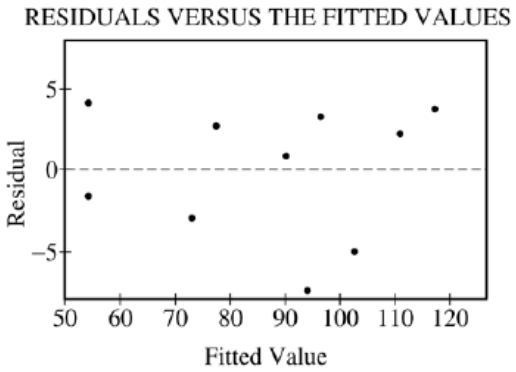
The Great Plains Railroad is interested in studying how fuel consumption is related to the number of railcars for its trains on a certain route between Oklahoma City and Omaha.

A random sample of 10 trains on this route has yielded the data in the table below.

Number of Railcars	Fuel Consumption (units/mile)
20	58
20	52
37	91
31	80
47	114
43	98
39	87
50	122
40	100
29	70

A scatterplot, a residual plot, and the output from the regression analysis for these data are shown below.





The regression equation is
Fuel Consumption = 10.7 + 2.15 Railcars

Predictor	Coef	StDev	T	P
Constant	10.677	5.157	2.07	0.072
Railcar	2.1495	0.1396	15.40	0.000

S = 4.361 R-Sq = 96.7% R-Sq(adj) = 96.3%

(a) Is a linear model appropriate for modeling these data? Clearly explain your reasoning.

(b) Suppose the fuel consumption cost is $25 per unit. Give a point estimate (single value) for the change in the average cost of fuel per mile for each additional railcar attached to a train. Show your work.

(c) Interpret the value of r^2 in the context of this problem.

(d) Would it be reasonable to use the fitted regression equation to predict the fuel consumption for a train on this route if the train had 65 railcars? Explain.