AP Calculus AB

Topic 8: Particle Motion

Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

(a) For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

(b) For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

(c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

(d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$.

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(d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$.

(a) $r'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$ $r'(t) = 0$ when $t = 1$ and $t = 3$ $r'(t) > 0$ for $0 < t < 1$ and $3 < t < 6$ $r'(t) < 0$ for $1 < t < 3$ Therefore R is moving to the right for $0 < t < 1$ and $3 < t < 6$.	$2: \begin{cases} 1: r'(t) \\ 1: \text{answer} \end{cases}$
(b) $p'(t) = -2 \cdot \frac{\pi}{4} \sin\left(\frac{\pi}{4}t\right)$ $p'(t) = 0$ when $t = 0$ and $t = 4$ $p'(t) < 0$ for $0 < t < 4$ $p'(t) > 0$ for $4 < t < 6$ Therefore the particles travel in opposite directions for $0 < t < 1$ and $3 < t < 4$.	$3: \begin{cases} 1: p'(t) \\ 1: \text{sign analysis for } p'(t) \\ 1: \text{answer} \end{cases}$
(c) $p''(t) = -2 \cdot \frac{\pi}{4} \cdot \frac{\pi}{4} \cos\left(\frac{\pi}{4}t\right)$ $p''(3) = -2\left(\frac{\pi}{4}\right)^2 \cos\left(\frac{3\pi}{4}\right) = \frac{\pi^2}{8} \cdot \frac{\sqrt{2}}{2} > 0$ $p'(3) < 0$ Therefore particle P is slowing down at time $t = 3$.	$2: \begin{cases} 1: p''(3) \\ 1: \text{answer with reason} \end{cases}$
(d) $\frac{1}{2}\int_1^3 \lvert p(t) - r(t) \rvert \, dt$	$2: \begin{cases} 1: \text{integrand} \\ 1: \text{limits and constant} \end{cases}$

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

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(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

(a) $v(5.5) = -0.45337$, $a(5.5) = -1.35851$ The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.	2 : conclusion with reason
(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) Distance $= \int_0^6 \lvert v(t) \rvert\,dt = 12.573$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$. $v(t)$ changes sign from positive to negative at time $t = b$. $x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135	$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$.

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

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(d) Find the position of the particle at time $t = 4$.

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \Rightarrow t = 3,\ 9$

The particle is moving to the left when $v(t) < 0$.
This occurs when $3 < t < 9$.

$2 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{interval} \end{cases}$

(b) $\int_0^6 |v(t)|\,dt$

1 : answer

(c) $a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

$a(4) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$

$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$

The speed is increasing at time $t = 4$, because velocity and acceleration have the same sign.

$3 : \begin{cases} 1 : a(t) \\ 2 : \text{conclusion with reason} \end{cases}$

(d)
$$\begin{aligned} x(4) &= -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt \\ &= -2 + \left[\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right]_0^4 \\ &= -2 + \frac{6}{\pi}\left[\sin\left(\frac{2\pi}{3}\right) - 0\right] \\ &= -2 + \frac{6}{\pi}\cdot\frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi} \end{aligned}$$

$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

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(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(a) Solve $\lvert v(t)\rvert = 2$ on $2 \le t \le 4$. $t = 3.128$ (or 3.127) and $t = 3.473$	$2 : \begin{cases} 1 : \text{considers } \lvert v(t)\rvert = 2 \\ 1 : \text{answer} \end{cases}$
(b) $s(t) = 10 + \int_0^t v(x)\,dx$ $s(5) = 10 + \int_0^5 v(x)\,dx = -9.207$	$2 : \begin{cases} 1 : s(t) \\ 1 : s(5) \end{cases}$
(c) $v(t) = 0$ when $t = 0.536033, 3.317756$ $v(t)$ changes sign from negative to positive at time $t = 0.536033$. $v(t)$ changes sign from positive to negative at time $t = 3.317756$. Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).	$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 2 : \text{answers with justification} \end{cases}$
(d) $v(4) = -11.475758 < 0,\ a(4) = v'(4) = -22.295714 < 0$ The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.	2 : conclusion with reason

For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the position of the particle at time $t = 0$.

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

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Solution	Scoring
(a) $v(4) = 2.978716 > 0$ $v'(4) = -1.164000 < 0$ The particle is slowing down since the velocity and acceleration have different signs.	2 : conclusion with reason
(b) $v(t) = 0 \Rightarrow t = 2.707468$ $v(t)$ changes from positive to negative at $t = 2.707$. Therefore, the particle changes direction at this time.	$2 : \begin{cases} 1 : t = 2.707 \\ 1 : \text{justification} \end{cases}$
(c) $x(0) = x(4) + \int_4^0 v(t)\,dt$ $= 2 + (-5.815027) = -3.815$	$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$
(d) Distance $= \int_0^3 \lvert v(t) \rvert\,dt = 5.301$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

A particle moves along the x-axis so that its velocity v at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

(a) On the axes provided, sketch the graph of $v(t)$ for $0 \le t \le 16$.

(b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

(d) Is there any time t, $0 < t \le 16$, at which the particle returns to the origin? Justify your answer.

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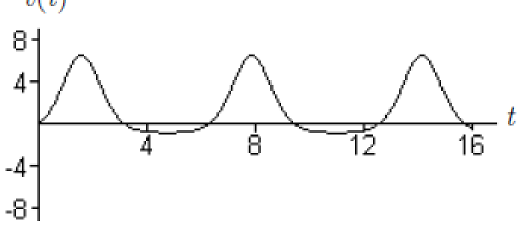
(a)



1 : graph

three "humps"

periodic behavior

starts at origin

reasonable relative max and min values

(b) Particle is moving to the left when

$v(t) < 0$, i.e. $e^{2\sin t} < 1$.

$(\pi, 2\pi)$, $(3\pi, 4\pi)$ and $(5\pi, 16]$

3 { 2 : intervals

$< -1 >$ each missing or incorrect interval

1 : reason

(c) $\int_0^4 |v(t)|\,dt = 10.542$

or

$v(t) = e^{2\sin t} - 1 = 0$

$t = 0$ or $t = \pi$

$x(\pi) = \int_0^\pi v(t)\,dt = 10.10656$

$x(4) = \int_0^4 v(t)\,dt = 9.67066$

$|x(\pi) - x(0)| + |x(4) - x(\pi)|$

$= 10.542$

3 { 1 : limits of 0 and 4 on an integral of

$v(t)$ or $|v(t)|$

or

uses $x(0)$ and $x(4)$ to compute distance

1 : handles change of direction at student's turning point

1 : answer

note: 0/1 if incorrect turning point

(d) There is no such time because

$\int_0^T v(t)\,dt > 0$ for all $T > 0$.

2 { 1 : no such time

1 : reason

An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

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(a) $a(4) = v'(4) = \frac{\pi}{3}\cos\left(\frac{4\pi}{3}\right)$

$= -\frac{\pi}{6}$ or -0.523 or -0.524

1 : answer

(b) On $3 < t < 4.5$:

$a(t) = v'(t) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right) < 0$

Statement I is correct since $a(t) < 0$.

Statement II is correct since $v(t) < 0$ and $a(t) < 0$.

3 { 1 : I correct, with reason
1 : II correct
1 : reason for II }

(c) Distance $= \int_0^4 |v(t)|\,dt = 2.387$

OR

$x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$

$x(0) = 2$

$x(4) = 2 + \frac{9}{2\pi} = 3.43239$

$v(t) = 0$ when $t = 3$

$x(3) = \frac{6}{\pi} + 2 = 3.90986$

$|x(3) - x(0)| + |x(4) - x(3)| = \frac{15}{2\pi} = 2.387$

3 { 1 : { limits of 0 and 4 on an integral of $v(t)$ or $|v(t)|$
or
uses $x(0)$ and $x(4)$ to compute distance }
1 : handles change of direction at student's turning point
1 : answer
0/1 if incorrect turning point or no turning point }

(d) $x(4) = x(0) + \int_0^4 v(t)\,dt = 3.432$

OR

$x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$

$x(4) = 2 + \frac{9}{2\pi} = 3.432$

2 { 1 : integral
1 : answer }

OR

2 { 1 : $x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + C$
1 : answer
0/1 if no constant of integration }

A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of t at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

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Solution	Scoring
(a) $a(t) = v'(t) = -e^{1-t}$ $a(3) = -e^{-2}$	$2 : \begin{cases} 1 : v'(t) \\ 1 : a(3) \end{cases}$
(b) $a(3) < 0$ $v(3) = -1 + e^{-2} < 0$ Speed is increasing since $v(3) < 0$ and $a(3) < 0$.	1 : answer with reason
(c) $v(t) = 0$ when $1 = e^{1-t}$, so $t = 1$. $v(t) > 0$ for $t < 1$ and $v(t) < 0$ for $t > 1$. Therefore, the particle changes direction at $t = 1$.	$2 : \begin{cases} 1 : \text{solves } v(t) = 0 \text{ to get } t = 1 \\ 1 : \text{justifies change in direction at } t = 1 \end{cases}$

(d) Distance $= \int_0^3 \lvert v(t) \rvert \, dt$

$$= \int_0^1 \left(-1 + e^{1-t}\right) dt + \int_1^3 \left(1 - e^{1-t}\right) dt$$

$$= \left(-t - e^{1-t} \Big|_0^1\right) + \left(t + e^{1-t} \Big|_1^3\right)$$

$$= (-1 - 1 + e) + \left(3 + e^{-2} - 1 - 1\right)$$

$$= e + e^{-2} - 1$$

$4 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{antidifferentiation} \\ 1 : \text{evaluation} \end{cases}$

OR

$x(t) = -t - e^{1-t}$

$x(0) = -e$

$x(1) = -2$

$x(3) = -e^{-2} - 3$

Distance $= (x(1) - x(0)) + (x(1) - x(3))$

$$= (-2 + e) + \left(1 + e^{-2}\right)$$

$$= e + e^{-2} - 1$$

$4 : \begin{cases} 1 : \text{any antiderivative} \\ 1 : \text{evaluates } x(t) \text{ when } t = 0, 1, 3 \\ 1 : \text{evaluates distance between points} \\ 1 : \text{evaluates total distance} \end{cases}$

A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(d) During the time interval $0 \leq t \leq 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

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(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

(a) $a(2) = v'(2) = 1.587$ or 1.588 $v(2) = -3\sin(2) < 0$ Speed is decreasing since $a(2) > 0$ and $v(2) < 0$.	$2: \begin{cases} 1: a(2) \\ 1: \text{speed decreasing with reason} \end{cases}$
(b) $v(t) = 0$ when $\frac{t^2}{2} = \pi$ $t = \sqrt{2\pi}$ or 2.506 or 2.507 Since $v(t) < 0$ for $0 < t < \sqrt{2\pi}$ and $v(t) > 0$ for $\sqrt{2\pi} < t < 3$, the particle changes directions at $t = \sqrt{2\pi}$.	$2: \begin{cases} 1: t = \sqrt{2\pi} \text{ only} \\ 1: \text{justification} \end{cases}$
(c) Distance $= \int_0^3 \lvert v(t)\rvert\, dt = 4.333$ or 4.334	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(d) $\int_0^{\sqrt{2\pi}} v(t)\, dt = -3.265$ $x(\sqrt{2\pi}) = x(0) + \int_0^{\sqrt{2\pi}} v(t)\, dt = -2.265$ Since the total distance from $t = 0$ to $t = 3$ is 4.334, the particle is still to the left of the origin at $t = 3$. Hence the greatest distance from the origin is 2.265.	$2: \begin{cases} 1: \pm \text{ (distance particle travels while velocity is negative)} \\ 1: \text{answer} \end{cases}$

A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

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(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

(a) $a(2) = v'(2) = -0.132$ or -0.133	1 : answer
(b) $v(2) = -0.436$ Speed is increasing since $a(2) < 0$ and $v(2) < 0$.	1 : answer with reason
(c) $v(t) = 0$ when $\tan^{-1}\left(e^t\right) = 1$ $t = \ln(\tan(1)) = 0.443$ is the only critical value for y. $v(t) > 0$ for $0 < t < \ln(\tan(1))$ $v(t) < 0$ for $t > \ln(\tan(1))$ $y(t)$ has an absolute maximum at $t = 0.443$.	3 : { 1 : sets $v(t) = 0$ 1 : identifies $t = 0.443$ as a candidate 1 : justifies absolute maximum
(d) $y(2) = -1 + \int_0^2 v(t)\,dt = -1.360$ or -1.361 The particle is moving away from the origin since $v(2) < 0$ and $y(2) < 0$.	4 : { 1 : $\int_0^2 v(t)\,dt$ 1 : handles initial condition 1 : value of $y(2)$ 1 : answer with reason

A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

(c) Find the position of the particle at time $t = 2$.

(d) Find the average speed of the particle over the interval $0 \le t \le 2$.

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(d) Find the average speed of the particle over the interval $0 \le t \le 2$.

(a) $a(4) = v'(4) = \frac{5}{7}$

1 : answer

(b) $v(t) = 0$

$t^2 - 3t + 3 = 1$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 1, 2$

$v(t) > 0$ for $0 < t < 1$

$v(t) < 0$ for $1 < t < 2$

$v(t) > 0$ for $2 < t < 5$

The particle changes direction when $t = 1$ and $t = 2$.
The particle travels to the left when $1 < t < 2$.

3 : { 1 : sets $v(t) = 0$; 1 : direction change at $t = 1, 2$; 1 : interval with reason }

(c) $s(t) = s(0) + \int_0^t \ln\left(u^2 - 3u + 3\right)\, du$

$s(2) = 8 + \int_0^2 \ln\left(u^2 - 3u + 3\right)\, du$

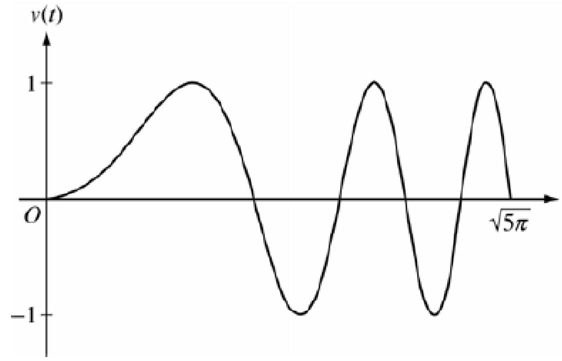
$= 8.368$ or 8.369

3 : { 1 : $\int_0^2 \ln\left(u^2 - 3u + 3\right)\, du$; 1 : handles initial condition; 1 : answer }

(d) $\frac{1}{2}\int_0^2 |v(t)|\, dt = 0.370$ or 0.371

2 : { 1 : integral; 1 : answer }

A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.



(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

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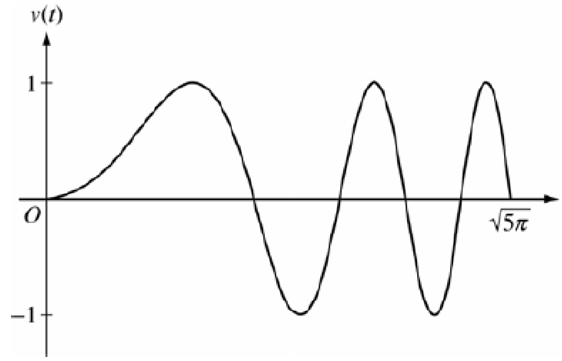

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(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \le t \le \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

(a) $a(3) = v'(3) = 6\cos 9 = -5.466$ or -5.467

1 : $a(3)$

(b) Distance $= \int_0^3 |v(t)|\, dt = 1.702$

OR

For $0 < t < 3$, $v(t) = 0$ when $t = \sqrt{\pi} = 1.77245$ and $t = \sqrt{2\pi} = 2.50663$

$x(0) = 5$

$x(\sqrt{\pi}) = 5 + \int_0^{\sqrt{\pi}} v(t)\, dt = 5.89483$

$x(\sqrt{2\pi}) = 5 + \int_0^{\sqrt{2\pi}} v(t)\, dt = 5.43041$

$x(3) = 5 + \int_0^3 v(t)\, dt = 5.77356$

$|x(\sqrt{\pi}) - x(0)| + |x(\sqrt{2\pi}) - x(\sqrt{\pi})| + |x(3) - x(\sqrt{2\pi})| = 1.702$

2 : { 1 : setup; 1 : answer }

(c) $x(3) = 5 + \int_0^3 v(t)\, dt = 5.773$ or 5.774

3 : { 2 { 1 : integrand; 1 : uses $x(0) = 5$ }; 1 : answer }

(d) The particle's rightmost position occurs at time $t = \sqrt{\pi} = 1.772$.

The particle changes from moving right to moving left at those times t for which $v(t) = 0$ with $v(t)$ changing from positive to negative, namely at $t = \sqrt{\pi}, \sqrt{3\pi}, \sqrt{5\pi}$ $(t = 1.772, 3.070, 3.963)$.

Using $x(T) = 5 + \int_0^T v(t)\, dt$, the particle's positions at the times it changes from rightward to leftward movement are:

T:	0	$\sqrt{\pi}$	$\sqrt{3\pi}$	$\sqrt{5\pi}$
$x(T)$:	5	5.895	5.788	5.752

The particle is farthest to the right when $T = \sqrt{\pi}$.

3 : { 1 : sets $v(t) = 0$; 1 : answer; 1 : reason }

A particle moves along the x-axis with position at time t given by $x(t) = e^{-t} \sin t$ for $0 \le t \le 2\pi$.

(a) Find the time t at which the particle is farthest to the left. Justify your answer.

(b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

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(a) Find the time t at which the particle is farthest to the left. Justify your answer.

(b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

(a) $x'(t) = -e^{-t} \sin t + e^{-t} \cos t = e^{-t}(\cos t - \sin t)$

$x'(t) = 0$ when $\cos t = \sin t$. Therefore, $x'(t) = 0$ on $0 \le t \le 2\pi$ for $t = \frac{\pi}{4}$ and $t = \frac{5\pi}{4}$.

The candidates for the absolute minimum are at $t = 0, \frac{\pi}{4}, \frac{5\pi}{4},$ and 2π.

t	$x(t)$
0	$e^0 \sin(0) = 0$
$\frac{\pi}{4}$	$e^{-\frac{\pi}{4}} \sin\left(\frac{\pi}{4}\right) > 0$
$\frac{5\pi}{4}$	$e^{-\frac{5\pi}{4}} \sin\left(\frac{5\pi}{4}\right) < 0$
2π	$e^{-2\pi} \sin(2\pi) = 0$

The particle is farthest to the left when $t = \frac{5\pi}{4}$.

$$5 : \begin{cases} 2 : x'(t) \\ 1 : \text{sets } x'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

(b) $x''(t) = -e^{-t}(\cos t - \sin t) + e^{-t}(-\sin t - \cos t)$

$= -2e^{-t} \cos t$

$Ax''(t) + x'(t) + x(t)$

$= A\left(-2e^{-t} \cos t\right) + e^{-t}(\cos t - \sin t) + e^{-t} \sin t$

$= (-2A + 1)e^{-t} \cos t$

$= 0$

Therefore, $A = \frac{1}{2}$.

$$4 : \begin{cases} 2 : x''(t) \\ 1 : \text{substitutes } x''(t),\ x'(t), \text{ and } x(t) \\ \quad \text{into } Ax''(t) + x'(t) + x(t) \\ 1 : \text{answer} \end{cases}$$

t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	–29	55	48

The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

(a) Justify why there must be at least one time t, for $0.3 \le t \le 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

(b) Use a trapezoidal sum with the three subintervals [0, 0.3], [0.3, 1.7], and [1.7, 2.8] to approximate the value of $\int_0^{2.8} v_P(t)\,dt$.

(c) A second particle, Q, also moves along the x-axis so that its velocity for $0 \le t \le 4$ is given by $v_Q(t) = 45\sqrt{t}\cos\left(0.063t^2\right)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.

(d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	−29	55	48

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(d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

(a) The average rate of change of velocity in the interval $0.3 \le t \le 2.8$ is zero, and because the function $v(t)$ is continuous and differentiable, by the Mean Value theorem there must be at least one point in time between $0.3 \le t \le 2.8$ when instantaneous velocity is zero. ■

(b) $\int_0^{2.8} v_p(t)\,dt \approx \frac{0+55}{2}(0.3-0) + \frac{55+(-29)}{2}(1.7-0.3) + \frac{(-29+55)}{2}(2.8-1.7) = 40.75$ meters ■

(c) Using technology, $v_Q(t)$ is at least 60 m/hour between

$1.866 < t \le 3.519$

Distance $= \int_{1.866}^{3.519} |v_Q(t)|\,dt = 106.109$ meters ■

(d) $x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t)\,dt = -90 + 135.938 = 45.938$

$x_P(2.8) = x_P(0) + 40.75 = 40.75$

[The distance between the particles is 5.188 meters at $t = 2.8$.] ■