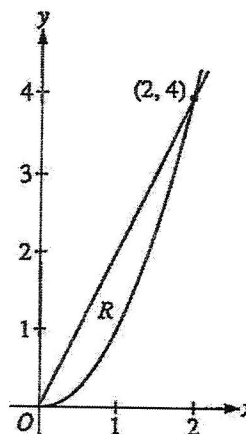


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Question 4

Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.



- (a) Find the area of R .
- (b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.
- (c) Another solid has the same base R . For this solid, the cross sections perpendicular to the y -axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

$$\begin{aligned} \text{(a) Area} &= \int_0^2 (2x - x^2) dx \\ &= x^2 - \frac{1}{3}x^3 \Big|_{x=0}^{x=2} \\ &= \frac{4}{3} \end{aligned}$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b) Volume} &= \int_0^2 \sin\left(\frac{\pi}{2}x\right) dx \\ &= -\frac{2}{\pi} \cos\left(\frac{\pi}{2}x\right) \Big|_{x=0}^{x=2} \\ &= \frac{4}{\pi} \end{aligned}$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

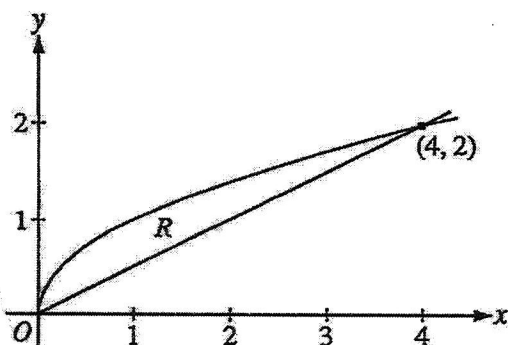
$$\text{(c) Volume} = \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \end{cases}$

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2009 SCORING GUIDELINES (Form B)

Question 4

Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.



- (a) Find the area of R .
- (b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are squares. Find the volume of this solid.
- (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

(a) Area = $\int_0^4 \left(\sqrt{x} - \frac{x}{2} \right) dx = \frac{2}{3} x^{3/2} - \frac{x^2}{4} \Big|_{x=0}^{x=4} = \frac{4}{3}$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume = $\int_0^4 \left(\sqrt{x} - \frac{x}{2} \right)^2 dx = \int_0^4 \left(x - x^{3/2} + \frac{x^2}{4} \right) dx$
 $= \frac{x^2}{2} - \frac{2x^{5/2}}{5} + \frac{x^3}{12} \Big|_{x=0}^{x=4} = \frac{8}{15}$

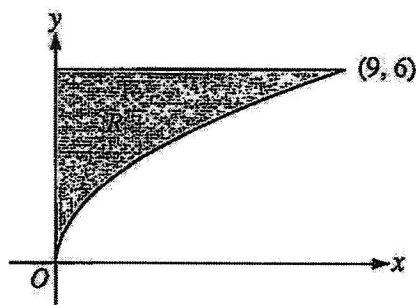
3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) Volume = $\pi \int_0^4 \left(\left(2 - \frac{x}{2} \right)^2 - (2 - \sqrt{x})^2 \right) dx$

3 : $\begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$

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Question 4



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y -axis, as shown in the figure above.

- Find the area of R .
- Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.
- Region R is the base of a solid. For each y , where $0 \leq y \leq 6$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose height is 3 times the length of its base in region R . Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) $\text{Area} = \int_0^9 (6 - 2\sqrt{x}) \, dx = \left(6x - \frac{4}{3}x^{3/2} \right) \Big|_{x=0}^{x=9} = 18$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $\text{Volume} = \pi \int_0^9 ((7 - 2\sqrt{x})^2 - (7 - 6)^2) \, dx$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

(c) Solving $y = 2\sqrt{x}$ for x yields $x = \frac{y^2}{4}$.

Each rectangular cross section has area $\left(3 \frac{y^2}{4} \right) \left(\frac{y^2}{4} \right) = \frac{3}{16}y^4$.

$\text{Volume} = \int_0^6 \frac{3}{16}y^4 \, dy$

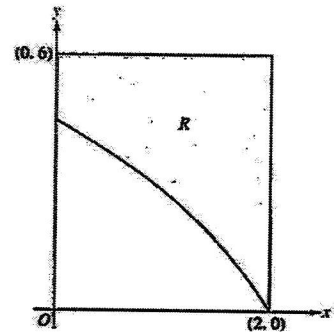
3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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2010 SCORING GUIDELINES (Form B)

Question 1

In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

- (a) Find the area of R .
 (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.
 (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of the solid.



(a) $\int_0^2 (6 - 4\ln(3 - x)) \, dx = 6.816 \text{ or } 6.817$

(b) $\pi \int_0^2 ((8 - 4\ln(3 - x))^2 - (8 - 6)^2) \, dx$
 $= 168.179 \text{ or } 168.180$

(c) $\int_0^2 (6 - 4\ln(3 - x))^2 \, dx = 26.266 \text{ or } 26.267$

1 : Correct limits in an integral in (a), (b), or (c)

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

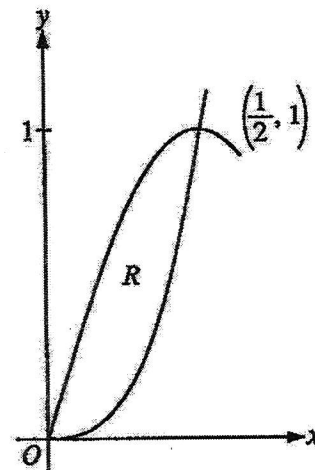
3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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Question 3

Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

- (a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.
- (b) Find the area of R .
- (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



(a) $f\left(\frac{1}{2}\right) = 1$

$f'(x) = 24x^2$, so $f'\left(\frac{1}{2}\right) = 6$

An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.

2: $\begin{cases} 1: f'\left(\frac{1}{2}\right) \\ 1: \text{answer} \end{cases}$

(b) Area = $\int_0^{1/2} (g(x) - f(x)) dx$
 $= \int_0^{1/2} (\sin(\pi x) - 8x^3) dx$
 $= \left[-\frac{1}{\pi} \cos(\pi x) - 2x^4 \right]_{x=0}^{x=1/2}$
 $= -\frac{1}{8} + \frac{1}{\pi}$

4: $\begin{cases} 1: \text{integrand} \\ 2: \text{antiderivative} \\ 1: \text{answer} \end{cases}$

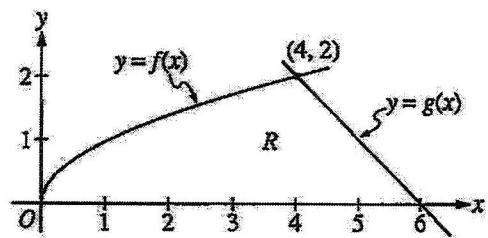
(c) $\pi \int_0^{1/2} ((1 - f(x))^2 - (1 - g(x))^2) dx$
 $= \pi \int_0^{1/2} ((1 - 8x^3)^2 - (1 - \sin(\pi x))^2) dx$

3: $\begin{cases} 1: \text{limits and constant} \\ 2: \text{integrand} \end{cases}$

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2011 SCORING GUIDELINES (Form B)

Question 3

The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x -axis and the graphs of f and g , as shown in the figure above.



- (a) Find the area of R .
- (b) The region R is the base of a solid. For each y , where $0 \leq y \leq 2$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.
- (c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g . Find the coordinates of point P .

(a) Area = $\int_0^4 \sqrt{x} \, dx + \frac{1}{2} \cdot 2 \cdot 2 = \frac{2}{3} x^{3/2} \Big|_{x=0}^{x=4} + 2 = \frac{22}{3}$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $y = \sqrt{x} \Rightarrow x = y^2$
 $y = 6 - x \Rightarrow x = 6 - y$

Width = $(6 - y) - y^2$

Volume = $\int_0^2 2y(6 - y - y^2) \, dy$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $g'(x) = -1$

Thus a line perpendicular to the graph of g has slope 1.

$f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{4}$

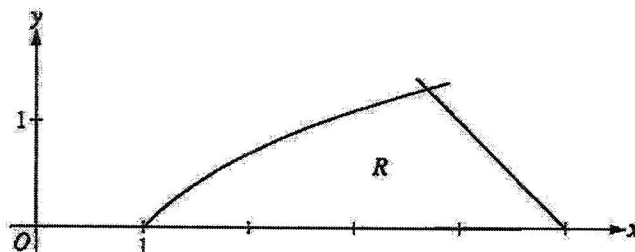
The point P has coordinates $\left(\frac{1}{4}, \frac{1}{2}\right)$.

3 : $\begin{cases} 1 : f'(x) \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

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Question 2

Let R be the region in the first quadrant bounded by the x -axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



- (a) Find the area of R .
- (b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
- (c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .

$$\ln x = 5 - x \Rightarrow x = 3.69344$$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

$$\begin{aligned} \text{(a) Area} &= \int_0^B (5 - y - e^y) dy \\ &= 2.986 \text{ (or } 2.985) \end{aligned}$$

OR

$$\begin{aligned} \text{Area} &= \int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx \\ &= 2.986 \text{ (or } 2.985) \end{aligned}$$

$$\text{(b) Volume} = \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$$

$$\text{(c) } \int_0^k (5 - y - e^y) dy = \frac{1}{2} \cdot 2.986 \text{ (or } \frac{1}{2} \cdot 2.985)$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

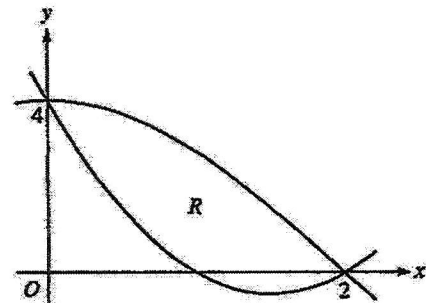
3 : $\begin{cases} 2 : \text{integrands} \\ 1 : \text{expression for total volume} \end{cases}$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{equation} \end{cases}$

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Question 5

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g , as shown in the figure above.



- (a) Find the area of R .
- (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.
- (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$$\begin{aligned} \text{(a) Area} &= \int_0^2 [g(x) - f(x)] dx \\ &= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - (2x^2 - 6x + 4) \right] dx \\ &= \left[4 \cdot \frac{4}{\pi} \sin\left(\frac{\pi}{4}x\right) - \left(\frac{2x^3}{3} - 3x^2 + 4x\right) \right]_0^2 \\ &= \frac{16}{\pi} - \left(\frac{16}{3} - 12 + 8\right) = \frac{16}{\pi} - \frac{4}{3} \end{aligned}$$

4 : $\begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2 \right] dx \\ &= \pi \int_0^2 \left[(4 - (2x^2 - 6x + 4))^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2 \right] dx \end{aligned}$$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

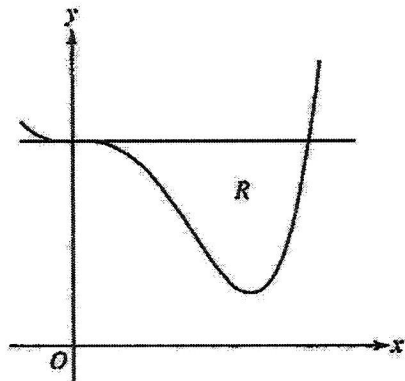
$$\begin{aligned} \text{(c) Volume} &= \int_0^2 [g(x) - f(x)]^2 dx \\ &= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - (2x^2 - 6x + 4) \right]^2 dx \end{aligned}$$

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

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Question 2

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.



- (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
- (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is an isosceles right triangle with a leg in R . Find the volume of the solid.
- (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k .

(a) $f(x) = 4 \Rightarrow x = 0, 2.3$

$$\begin{aligned} \text{Volume} &= \pi \int_0^{2.3} [(4+2)^2 - (f(x)+2)^2] dx \\ &= 98.868 \text{ (or } 98.867) \end{aligned}$$

4 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) $\text{Volume} = \int_0^{2.3} \frac{1}{2} (4 - f(x))^2 dx$
 $= 3.574 \text{ (or } 3.573)$

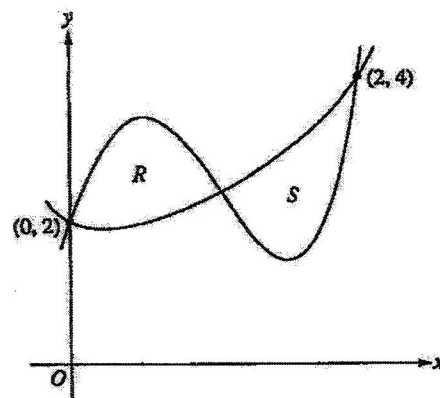
3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^k (4 - f(x)) dx = \int_k^{2.3} (4 - f(x)) dx$

2 : $\begin{cases} 1 : \text{area of one region} \\ 1 : \text{equation} \end{cases}$

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

- (a) Find the sum of the areas of regions R and S .
- (b) Region S is the base of a solid whose cross sections perpendicular to the x -axis are squares. Find the volume of the solid.
- (c) Let h be the vertical distance between the graphs of f and g in region S . Find the rate at which h changes with respect to x when $x = 1.8$.



- (a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$.

$$\begin{aligned} \text{Area} &= \int_0^A [g(x) - f(x)] dx + \int_A^2 [f(x) - g(x)] dx \\ &= 0.997427 + 1.006919 = 2.004 \end{aligned}$$

- (b) $\text{Volume} = \int_A^2 [f(x) - g(x)]^2 dx = 1.283$

- (c) $h(x) = f(x) - g(x)$
 $h'(x) = f'(x) - g'(x)$
 $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811)

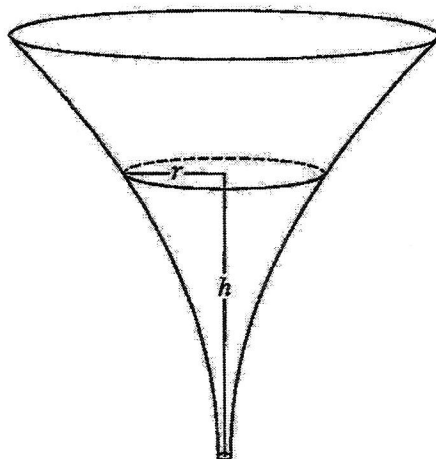
$$4 : \begin{cases} 1 : \text{limits} \\ 2 : \text{integrands} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{considers } h' \\ 1 : \text{answer} \end{cases}$$

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Question 5



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.

- Find the average value of the radius of the funnel.
- Find the volume of the funnel.
- The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

$$\begin{aligned} \text{(a) Average radius} &= \frac{1}{10} \int_0^{10} \frac{1}{20}(3 + h^2) dh = \frac{1}{200} \left[3h + \frac{h^3}{3} \right]_0^{10} \\ &= \frac{1}{200} \left(\left(30 + \frac{1000}{3} \right) - 0 \right) = \frac{109}{60} \text{ in} \end{aligned}$$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^{10} \left(\left(\frac{1}{20} \right) (3 + h^2) \right)^2 dh = \frac{\pi}{400} \int_0^{10} (9 + 6h^2 + h^4) dh \\ &= \frac{\pi}{400} \left[9h + 2h^3 + \frac{h^5}{5} \right]_0^{10} \\ &= \frac{\pi}{400} \left(\left(90 + 2000 + \frac{100000}{5} \right) - 0 \right) = \frac{2209\pi}{40} \text{ in}^3 \end{aligned}$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(c) } \frac{dr}{dt} &= \frac{1}{20}(2h) \frac{dh}{dt} \\ \frac{1}{5} &= \frac{3}{10} \frac{dh}{dt} \\ \frac{dh}{dt} &= -\frac{1}{5} \cdot \frac{10}{3} = -\frac{2}{3} \text{ in/sec} \end{aligned}$$

3 : $\begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$